

# Electoral Ambiguity and Political Representation\*

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## Abstract

We introduce a Downsian model in which policy-relevant information is revealed to the elected politician after the election. The electorate benefits from giving the elected politician discretion to adapt policies to his information. But limits on discretion are desirable when politicians do not share the electorate's policy preferences. Optimal political representation generally consists of a mixture of the delegate (no discretion) and trustee (full discretion) models. Ambiguous electoral platforms are essential for achieving beneficial representation. Nevertheless, electoral competition does not ensure optimal representation: the winning candidate's platform is generally overly ambiguous. While there is a positive correlation between ambiguity and electoral success, the relationship is not causal.

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# 1. Introduction

It is common wisdom that “[p]oliticians are notoriously reluctant to take clear stands on the issues of the day” (Page, 1976, p. 742). Indeed, during the 2012 American Presidential election, media outlets routinely criticized the ambiguity of both Barack Obama’s and Mitt Romney’s campaign promises (Condon, 2012; Lizza, 2012). Even more recently, in the 2015 British general election, the non-partisan Institute for Fiscal Studies described parties’ manifestos as follows: “Where benefit cuts are proposed, they are largely unspecified (Conservatives), vague (Liberal Democrats) or trivially small relative to the rhetoric being used (Labour).”

While many scholars take a dim view of electoral ambiguity, we argue in this paper that ambiguity is central to beneficial political representation. Our approach builds on the Downsian framework, with one key twist. We posit that some information relevant to policy-making is revealed to the elected politician only after the election. We allow candidates, who are both office and policy motivated, to campaign by announcing a *policy set*. In other words, each candidate commits to a set of policies from which he will choose if elected. A candidate can propose a single policy (such as a precise figure for benefit cuts), commit to avoiding extreme policies (such as a bound on benefit cuts), or could announce more complicated sets (such as either a significant benefit cut or none at all).

Electoral ambiguity—i.e., not committing to a single policy—affords politicians discretion to adapt policies to new information.<sup>1</sup> While ambiguity creates uncertainty about which policy will be implemented, it can benefit voters if policies are better tailored to circumstances. However, whenever politicians do not share voters’ policy preferences, voters also benefit from constraints on discretion that mitigate post-electoral policy bias.<sup>2</sup> Our framework thus contributes to the classic question, dating back to James Madison and Edmund Burke, of whether political representation should be by delegates or trustees. In our account, voters seek a *mixture* of the delegate (no discretion) and trustee (full discretion) models of political representation.

Actual political representation, however, is determined by politicians’ strategic platform choices. Candidates would like maximum discretion, but they must propose platforms that are attractive enough to the electorate—specifically, the representative or median voter (hence-

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<sup>1</sup> Thus, our notion of ambiguity is that at the time of the election, voters are uncertain about what policy an elected politician will implement. This notion follows Downs (1957), Shepsle (1972), and other authors. There are more selective notions of ambiguity, such as Knightian uncertainty (Knight, 1921).

<sup>2</sup> Our model allows for any degree of disagreement over policy between candidates and voters; for ease of exposition, this introduction focuses on positive but limited disagreement.

forth “the voter”)—relative to their opponent’s. We show that candidates’ equilibrium platforms take a simple form: they retain some discretion but limit how far they can move policies in the direction of their bias. Actual representation thus resembles optimal representation. There is, however, a critical difference. The winning candidate is *overly ambiguous* from the voter’s point of view unless candidates’ policy preferences are symmetrically biased relative to the electorate. When candidates are asymmetric, the less-biased candidate wins the election while exploiting his greater policy alignment to grant himself more discretion than is optimal for the voter. So, while electoral competition disciplines the winning candidate, it is not generally sufficient to ensure voter-optimal outcomes.

As predicted by our theory, one often observes candidates reassuring voters that their policies would not be too extreme without spelling out exactly what they will do if elected. Examples include the U.K. Conservative party recently promising to increase funding for the Department of Health by at least £8 billion; Mitt Romney pledging in 2012 that his reform of social security would entail “no change for those at or near retirement” (Sahadi, 2012); and Barack Obama guaranteeing in 2008 that “no family making less than \$250,000 a year will see any form of tax increase” (Obama, 2008).

Furthermore, consistent with empirical evidence (Berensky and Lewis, 2007; Tomz and Van Houweling, 2009), we find that ambiguity is not punished by the electorate; to the contrary, the winning candidate is generally more ambiguous than his opponent. It bears emphasis that a candidate does not win *because* he is more ambiguous than his opponent; rather he is both more ambiguous *and* more likely to win because he already possesses an electoral advantage of greater preference alignment. The electoral success of ambiguous candidates has long puzzled political scientists; Downs (1957) himself noted that “candidates [in the real world] becloud their policies in a fog of ambiguity” despite the fact that ambiguity does not arise in standard specifications of his framework. Some theoretical explanations rely on voters’ risk preferences (Zeckhauser, 1969; Shepsle, 1972; Aragones and Postlewaite, 2002) or “behavioral” characteristics (Berliant and Konishi, 2005; Callander and Wilson, 2008; Jensen, 2009). Our results, which are obtained with risk-averse voters, harken back to the dictum that correlation does not imply causation.

Our model also generates policy divergence: the two candidates propose different platforms and induce different expected final policies, tilted towards their own policy preferences. Divergence occurs even when candidates are symmetrically biased and propose platforms that are optimal for the voter. In this case, the voter is indifferent between the two candidates and the election can then be viewed as close. Consistent with the empirical results of Lee,

Moretti, and Butler (2004), we find that divergence emerges even in close elections. Notably, in our model the electorate is not merely electing policies; it is genuinely affecting them.

There is a non-monotonic relationship between policy divergence and political polarization (the difference in candidates' preferences). Fixing platforms, greater political polarization leads to greater policy divergence. But platforms also respond to political polarization. As a candidate's policy preferences move further away from the voter's, electoral pressures induce him to propose less ambiguous platforms: to appeal to the voter, he offers himself less discretion. We find that, on balance, policy divergence is increasing in political polarization when polarization is low and decreasing when polarization is high. Indeed, when candidates' policy preferences are very similar to the voter's, they obviously choose approximately the same policy and policy divergence disappears. In contrast, as political polarization gets sufficiently large, policy divergence vanishes because candidates' platforms converge to the voter's ex-ante preferred policy. One implication is that a given empirical observation of policy divergence could be consistent with multiple levels of underlying political polarization, complicating identification of the level of political polarization.

We conclude this introduction by connecting our work to the most closely related literature. Meirowitz (2005) argues that both candidates and voters from their own party may desire ambiguity—modeled as not announcing a position—in a primary election; this preserves a candidate's flexibility to adjust his position in the subsequent general election to new information about the location of the whole electorate's median voter.<sup>3</sup> While there are some formal similarities, our substantive focus on political representation is entirely different and our modeling of electoral ambiguity is richer. By using policy sets to capture candidates' ambiguity, we follow Aragonés and Neeman (2000). In their model candidates value ambiguity while voters do not; see also Alesina and Cukierman (1990), Westermarck (2004), and Frenkel (2014). In our model all actors benefit from some electoral ambiguity because of a desire to tailor policy to an initially-uncertain “state of the world.” How much ambiguity is desired by the electorate depends on parameters such as candidates' preference alignment and uncertainty about the state.

Our work is related to the literature on delegation by a principal to a better-informed agent, pioneered in economics by Holmström (1977, 1984) and adopted for the study of the bureaucracy by Epstein and O'Halloran (1994) and others. Notable references include Melumad and Shibano (1991), Amador, Werning, and Angeletos (2006), and Amador and

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<sup>3</sup> Glazer (1990) and Kamada and Sugaya (2014) are other papers studying candidates' ambiguity that stems from uncertainty about voters' preferences.

Bagwell (2013) in economics and Bendor and Meirowitz (2004), Gailmard (2009), and Wiseman (2009) in political science. We specifically build on technical results of Alonso and Matouschek (2008), who derive conditions for the optimality of interval delegation. In contrast to the aforementioned papers, our framework has candidates (multiple agents) strategically proposing platforms (delegation sets) to the voter (principal). We are not aware of any extant literature on such “delegation games.” We establish conditions under which interval policy sets emerge even in our more complex strategic environment, and use this result to derive new insights about electoral competition and political representation.

There is virtually no formal work addressing the delegate-trustee trade-off in political representation.<sup>4</sup> To our knowledge, the only exception is Fox and Shotts (2009), who show how either form of representation may emerge when voters are uncertain about politicians’ preferences and competence. Unlike our interest in electoral competition, theirs is a model of political accountability. Fox and Shotts (2009) also do not consider any intermediate cases between a full delegate and a full trustee relationship. Our theory shows that optimal political representation genuinely spans the entire spectrum between the two models.

The rest of the paper is organized as follows. Section 2 introduces our formal model. Section 3 characterizes optimal platforms from the voter’s perspective. Section 4 contains our main results on equilibrium platforms and their implications. In Section 5 we discuss how alternative modeling choices. Section 6 concludes. Formal proofs of all results and some additional technical material are collected in the Appendices.

## 2. Model

We develop a model in the tradition of Downs (1957). There are two political candidates,  $L$  (left) and  $R$  (right), and a single representative voter, denoted  $0$ .<sup>5</sup> The game form is as follows:

1. Each candidate  $i \in \{L, R\}$  simultaneously chooses a *platform*  $A_i \subseteq \mathbb{R}$ .
2. The voter observes both platforms and elects one of the candidates,  $e \in \{L, R\}$ .
3. Nature determines a *state* of the world,  $\theta \in [-1, 1]$ , which is privately observed by the elected candidate  $e$ .
4. The elected candidate chooses a *policy* action,  $a \in A_i$ .

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<sup>4</sup> There is, of course, a massive literature in political theory on representation to which we cannot do justice here; see Urbinati and Warren (2008) for a review.

<sup>5</sup> The focus on a single voter is for ease of exposition; we consider in Subsection 5.1 a heterogenous electorate in which there is a decisive median voter.

We assume that candidates have commitment power: the final policy must be consistent with the candidate's platform. Crucially, the impact of any policy depends on a state of the world that is privately revealed to the elected politician only *after* the election but before his policy choice.<sup>6</sup> The state represents changing circumstances or new information that becomes available to a politician at the time of actual policy-making. As David Cameron stated to justify his child benefit cuts in an ITV interview in 2010, "We did not outline all those cuts, we did not know exactly the situation we were going to inherit."

We allow candidates to choose platforms that are *sets* of policies. This approach generalizes the standard model, as candidates can always propose a single policy (such as a precise figure for benefit cuts). In practice, of course, candidates are not constrained to be specific during their electoral campaigns, and they rarely choose to be. Notice that we have precluded candidates from committing to policies as a function of the state. This is justified because the state is privately observed by the office-holder; the voter only observes the chosen policy. State-contingent promises during the election would generally not be credible. See [Subsection 5.2](#) for further discussion.

**Preferences.** Each player  $i \in \{L, R, 0\}$  has policy preferences in state  $\theta$  that are single-peaked around a bliss point  $\alpha_i(\theta) := b_i + \theta \in \mathbb{R}$ . Specifically, player  $i$ 's policy utility is represented by the quadratic-loss utility function  $-(a - \theta - b_i)^2$ , where  $a$  is the policy.<sup>7</sup> We normalize  $b_0$  to 0, and to capture candidates on opposite sides of the voter, we assume  $b_L \leq 0 \leq b_R$ . For  $i \in \{L, R\}$ ,  $|b_i|$  measures the magnitude of preference (mis)alignment of candidate  $i$  with the voter. We refer to a candidate with  $b_i = 0$  as *unbiased*, a candidate with  $b_i \neq 0$  as *biased*, and a candidate with  $|b_i| \geq 1$  as *extreme*. As the state lies in  $[-1, 1]$ , an extreme candidate's bliss point is either always positive or always negative no matter the state.

Let  $e \in \{L, R\}$  denote the winner of the election. The voter only cares about policy, so her overall utility function is simply

$$u_0(a, \theta, e) := -(a - \theta)^2.$$

Candidates are both policy and office motivated as in [Wittman \(1977, 1983\)](#) and [Calvert \(1985\)](#).

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<sup>6</sup> By contrast, [Martinelli and Matsui \(2002\)](#), [Heidhues and Lagerlof \(2003\)](#), [Kartik, Squintani, and Tinn \(2014\)](#), and [Ambrus, Baranovskyi, and Kolb \(2015\)](#) consider models in which politicians are privately informed about a policy-relevant state *prior* to the election.

<sup>7</sup> As usual (e.g., [Gilligan and Krehbiel, 1987](#)), one can also view players as having state-independent preferences over outcomes or consequences; the state affects how policies map into outcomes.

Specifically, candidate  $i \in \{L, R\}$  has the utility function

$$u_i(a, \theta, e) := \begin{cases} \phi - (a - b_i - \theta)^2 & \text{if } e = i, \\ -(a - b_i - \theta)^2 & \text{if } e \neq i. \end{cases} \quad (1)$$

The parameter  $\phi > 0$  measures the degree of office motivation.

**Technical assumptions.** The state  $\theta \in [-1, 1]$  is distributed according to a cumulative distribution function (CDF)  $F$  that admits a differentiable density  $f$ . We assume  $f$  is positive and symmetric (i.e.,  $f(\theta) = f(-\theta) > 0$ ), and that it does not change too fast:

$$\text{for all } \theta \in [-1, 1] : -f(\theta) \leq f'(\theta) \leq f(\theta). \quad (2)$$

Intuitively, condition (2) requires sufficiently diffuse beliefs. The condition is likely to be satisfied in a policy domain where uncertainty about the appropriate policy is significant, such as an economic or foreign-policy issue. On the other hand, the condition is less appropriate for a social issue like abortion or gay rights.

We further impose that conditional expectations from tail truncations do not increase too fast:

$$\text{for all } t \in [-1, 1] : \frac{d}{dt} \mathbb{E}[\theta | \theta \geq t] < 1 \text{ and } \frac{d}{dt} \mathbb{E}[\theta | \theta \leq t] < 1. \quad (3)$$

Requirement (3) is satisfied by all log-concave densities, which covers a number of familiar distributions (Bagnoli and Bergstrom, 2005). Both the uniform distribution and the truncated normal distribution with mean zero and variance that is not too small satisfy all our requirements; see Remark 1 in Appendix A. Requirements (2) and (3) ensure a transparent characterization of delegation sets; we clarify the precise role subsequently (in particular, see fn. 11).

Finally, we require that each candidate  $i$ 's platform choice  $A_i$  must be a closed set, to ensure the office-holder's problem of choosing which policy to implement is well-behaved.

**Solution Concept.** All aspects of the model except the realization of the state  $\theta$  are common knowledge, and players are expected-utility maximizers. Our solution concept is Subgame Perfect Nash Equilibrium, hereafter simply *equilibrium*. (Standard refinements would not alter our results.) Notice that because  $\theta$  is observed only after the election, the office-holder's policymaking stage and the stage at which the voter chooses whom to elect both constitute proper subgames. Thus, an equilibrium satisfies the following properties: (i) if candidate  $i$  is



elected with platform  $A_i$ , in each state  $\theta$  he chooses  $a \in A_i$  to maximize  $u_i(\cdot)$ ;<sup>8</sup> (ii) taking (i) as given, for any  $A_L$  and  $A_R$  the voter elects the candidate who gives her the highest expected utility; (iii) taking (i) and (ii) as given, each candidate  $i$  proposes a platform  $A_i$  that maximizes his expected utility.<sup>9</sup>

**Terminology.** We say that a platform  $A_i$  is *minimal* if it contains no redundant policy: every  $a \in A_i$  will be chosen in some state if  $i$  is elected. To simplify the exposition and without any real loss of generality, we restrict attention to equilibria in which candidates use minimal platforms.<sup>10</sup> We say that (platform) *convergence* occurs if both candidates propose the same platform:  $A_L = A_R$ . This is a weak notion of convergence because it does not imply that both candidates implement the same policy if elected. Indeed, final policies coincide in all states only if either the candidates converge to a singleton platform ( $A_L = A_R = \{a\}$  for some  $a \in \mathbb{R}$ ) or they share the same policy preferences ( $b_L = b_R = 0$ ). When  $A_i$  is not a singleton, we say that candidate  $i$ 's platform is *ambiguous*. In other words, we view a candidate as ambiguous when the voter is uncertain about what policy will be implemented after the election, as argued by Page (1976). This perspective on ambiguity is shared by many others in the literature, including Downs (1957), Shepsle (1972), Aragones and Neeman (2000), and Aragones and Postlewaite (2002).

Finally, we view an *equilibrium's outcome* as its (distribution over the) winning candidate and its (possibly stochastic) mapping from states to final policies. In particular, two outcome-equivalent equilibria provide the same expected payoff to all players.

To wrap up this section, we would like to highlight that absent the unknown state of the world, our model satisfies all the assumptions of the Downsian framework (with policy motivated candidates): two candidates, unidimensional policy space, strictly concave utility functions, no uncertainty about voters' and candidates' preferences, and commitment by candidates to their platforms. Consequently, convergence would emerge if there were no uncertainty about the state of the world, or, as shown in Subsection 5.2, if platforms were constrained to single policies. With uncertainty about the state and ambiguity, however, candidates with different preferences will not always choose the same policy even if elected with the same platform. This feature generally precludes candidates from perfectly mimicking

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<sup>8</sup> As the state is continuously distributed, an "almost all" qualifier is relevant here and elsewhere; we omit this technicality to ease the exposition.

<sup>9</sup> Our analysis covers the possibility that candidates may mix over their platforms; as established in Lemma 1 below, candidates will not mix in equilibrium except possibly when they never win the election.

<sup>10</sup> Dropping this restriction creates some additional but irrelevant multiplicity: the additional equilibria that emerge are outcome-equivalent to those we focus on.



each other, unlike in most models of electoral competition.

### 3. Optimal Political Representation

We first characterize candidates' optimal platforms from the voter's perspective. In other words, we identify what platform the voter would like to endow each candidate  $i \in \{L, R\}$  with if  $i$  is going to be the office-holder. We call such a platform candidate  $i$ 's *voter-optimal platform*. A singleton policy platform is not necessarily voter-optimal because the voter would like the policy to be adapted ex-post to the state of the world. On the other hand, giving full discretion need not be optimal either because of the candidate's policy bias.

**Proposition 1** below describes how the voter resolves this trade-off between policy adaptability and policy bias. Recall that  $\alpha_i(\theta) = b_i + \theta$  denotes candidate  $i$ 's ideal policy in state  $\theta$ . It is convenient to define  $\bar{a}^0 \in [0, 1]$  and  $\underline{a}^0 \in [-1, 0]$  as the solutions to

$$\bar{a}^0 = \mathbb{E}[\theta | \theta \geq \bar{a}^0 - b_R] \quad \text{and} \quad \underline{a}^0 = \mathbb{E}[\theta | \theta \leq \underline{a}^0 - b_L]. \quad (4)$$

Condition (3) ensures that  $\bar{a}^0$  and  $\underline{a}^0$  are uniquely defined. Furthermore,  $\bar{a}^0 \leq b_R + 1$  ( $\underline{a}^0 \geq b_L - 1$ ) with equality if and only if  $b_R = 0$  ( $b_L = 0$ ), and  $\bar{a}^0 = 0$  ( $\underline{a}^0 = 0$ ) whenever  $b_R \geq 1$  ( $b_L \leq -1$ ). The symmetry of the state distribution also implies that  $\bar{a}^0 = -\underline{a}^0$  whenever  $b_R = -b_L$ .

To interpret (4), consider placing a ceiling  $\bar{a}^0$  on candidate  $R$ 's policy choice.  $R$  will then choose  $\bar{a}^0$  whenever the state is larger than  $\bar{a}^0 - b_R$ . The voter's preferred policy given  $\theta \geq \bar{a}^0 - b_R$  is simply  $\mathbb{E}[\theta | \theta \geq \bar{a}^0 - b_R]$ . Thus, the first equality in (4) requires that, on average, the voter receives her preferred policy when the policy ceiling on  $R$  binds.

**Proposition 1.** *Candidate  $R$ 's voter-optimal platform is*

$$A_R^0 := \begin{cases} \{0\} & \text{if } b_R \geq 1, \\ [\alpha_R(-1), \bar{a}^0] & \text{if } b_R \in [0, 1). \end{cases}$$

*Symmetrically, candidate  $L$ 's voter-optimal platform is*

$$A_L^0 := \begin{cases} \{0\} & \text{if } b_L \leq -1, \\ [\underline{a}^0, \alpha_L(1)] & \text{if } b_L \in (-1, 0]. \end{cases}$$

**Proposition 1** says a candidate's voter-optimal platform is a (possibly degenerate) interval. Technically, the problem of finding a voter-optimal platform is the same as that of a princi-

pal optimally deciding how much discretion to grant an informed agent with biased preferences. We do not require the voter to grant the politician an interval of policies to choose from. Indeed, in general, such interval delegation sets need not be optimal.<sup>11</sup> However, as the proof of [Proposition 1](#) establishes, our assumptions on the state distribution—in particular, requirements (2) and (3)—satisfy [Alonso and Matouschek’s \(2008\)](#) more general conditions guaranteeing optimality of interval delegation sets for any bias.

The optimal intervals in [Proposition 1](#) resolve the fundamental trade-off the voter faces between policy adaptability and the resulting policy bias (or, as it is sometimes referred to, policy drift). Plainly, when a politician is unbiased ( $b_i = 0$ ), it is voter-optimal to impose no constraints, as there is no trade-off. When a politician is extreme ( $|b_i| \geq 1$ ), the cost from policy bias is large relative to the gain from policy adaptability, and so the voter-optimal platform is a singleton: her ex-ante preferred policy,  $\{0\}$ . When a politician is biased but moderate ( $|b_i| \in (0, 1)$ ), it is optimal for the voter to endow him with some but not full discretion.

Candidate  $R$ ’s preferred policy is above the voter’s preferred policy in any state. There is thus no benefit to the voter of imposing a minimum policy on  $R$ . However, she may want to limit how far to the right  $R$  can push policy.  $R$ ’s voter-optimal platform is thus an interval from  $\alpha_R(-1)$ , the lowest policy  $R$  would ever take, up to a ceiling. Calculus determines the optimal ceiling to be  $\bar{a}^0$ , defined in [Equation 4](#). The reasoning is reversed for candidate  $L$ , who prefers policies lower than the voter in each state:  $L$ ’s voter-optimal platform is an interval with a floor on  $L$ ’s policy options.

Interval delegation, while common in the study of the bureaucracy ([Gailmard and Patty, 2012](#)), has not received attention in electoral politics. It has the intuitive property that the voter would like a politician to utilize his expertise but not distort policies excessively towards his own preferences. In our theory, then, optimal political representation takes a particular form: a *mixture* of the trustee model (full discretion) and the delegate model (no discretion). In fact, both models arise only as special cases. The trustee model is optimal when the politician is unbiased while the delegate model is optimal when the politician is extreme. Notice that, unless a politician is extreme, ambiguous platforms are necessary to attain optimal representation.

[Proposition 1](#) implies interesting comparative statics, which we summarize next. Let  $a_i(\theta, A)$  denote the policy chosen by candidate  $i$  in state  $\theta$  when elected with platform  $A$ ,

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<sup>11</sup> Here is a simple example in which interval delegation would not be optimal: the state has a binary support  $\{-1, 1\}$  and candidate  $i$ ’s bias satisfies  $|b_i| \in (0, 1)$ . In this case, any interval platform from candidate  $i$  is suboptimal for the voter; the (minimal) voter-optimal platform is  $\{-1, 1\}$ . While the example uses a binary support, the same point could be made using continuous distributions approximating binary support; such distributions would violate requirements (2) and (3).

and let  $W_j(A, i)$  be the expected utility of player  $j \in \{0, L, R\}$  when candidate  $i \in \{L, R\}$  is elected with platform  $A \subseteq \mathbb{R}$ .<sup>12</sup>

**Proposition 2.** *For any  $i \in \{L, R\}$  and  $b_i$  with  $|b_i| \in (0, 1)$ :*

1.  $A_i^0$  is decreasing (in the sense of set inclusion) in  $|b_i|$ .<sup>13</sup>
2.  $W_0(A_i^0, i)$  is decreasing in  $|b_i|$ ;
3.  $\mathbb{E}[a_L(\theta, A_L^0)] < 0 < \mathbb{E}[a_R(\theta, A_R^0)]$ , with  $\lim_{b_i \rightarrow 0} \mathbb{E}[a_i(\theta, A_i^0)] = \lim_{|b_i| \rightarrow 1} \mathbb{E}[a_i(\theta, A_i^0)] = 0$ .

**Proposition 2** focuses on the case where politicians are neither unbiased nor extreme, because those cases are trivial. Part 1 says that when a politician is more biased it is optimal to grant him less discretion, because the gain from limiting policy bias increases. Part 2 of **Proposition 2** says that, unsurprisingly, the voter is worse off when a politician is more biased, even under the voter-optimal platform. Finally, part 3 says that a politician's policy choice given the voter-optimal platform is tilted in expectation towards his own policy preferences. However, the *average policy bias*—as measured by  $\mathbb{E}[a_i(\theta, A_i^0)]$ , since the voter's ideal policy is on expectation 0—is non-monotonic in the politician's bias. When a politician is unbiased, he is given full discretion and chooses the voter's ideal policy in each state, which obviously induces no average policy bias. When a politician is extreme, average policy bias again vanishes, but now because the politician is optimally given no discretion; he is constrained to only choose the voter's ex-ante preferred policy, 0. Average policy bias emerges optimally only when a politician is moderately biased.

**Figure 1** summarizes some aspects of **Proposition 1** and **Proposition 2** by depicting the voter-optimal policy ceiling for candidate  $R$  as well as the corresponding expected policy.

## 4. Equilibrium Ambiguity and Political Representation

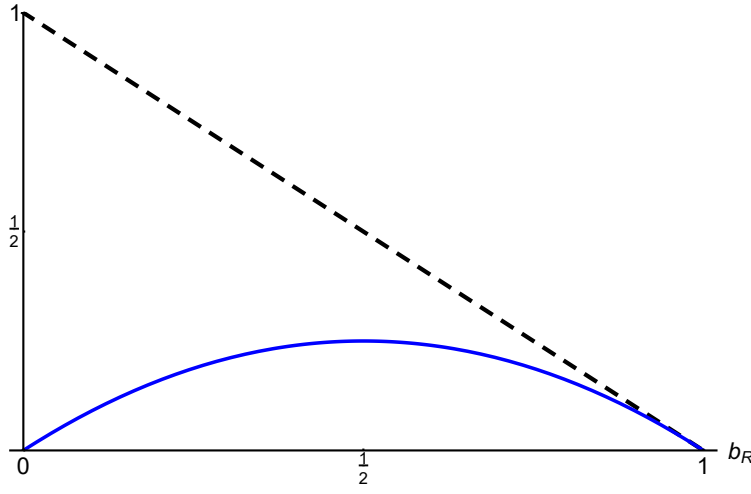
While **Proposition 1** and **Proposition 2** characterize properties of the voter-optimal platform, the voter does not actually choose a candidate's platform; rather, each candidate is free to propose any platform he would like. Plainly, conditional on being elected, a candidate always benefits from greater discretion (a larger  $A_i$  in the sense of set inclusion). The only

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<sup>12</sup> Explicitly:  $a_i(\theta, A) \in \arg \min_{a \in A} (a - b_i - \theta)^2$ ; if there are multiple minimizers, any one can be chosen. Also,

$W_j(A_i, i) := \phi \mathbb{I}_{\{i=j\}} - \int_{-1}^1 (a_i(\theta, A_i) - b_j - \theta)^2 f(\theta) d\theta$ , where  $\mathbb{I}$  denotes the indicator function.

<sup>13</sup> Throughout the paper, “decreasing” without a qualifier means “strictly decreasing”, and analogously for “increasing” and “preferred.”



**Figure 1** – Candidate  $R$ 's voter-optimal ceiling (black dashed line) and resulting expected policy (blue solid curve) as a function of  $b_R$ , given a uniform distribution of the state.

reason for him to propose a platform that limits his discretion is to secure office, both because he values office *per se* and in order to avoid his opponent tilting policies away from his preferences. In this section, we study how the office-holder's equilibrium discretion emerges from electoral competition and the strategic interplay between candidates.

Candidates are essentially unconstrained in their platform choices. Nevertheless, a winning candidate's equilibrium platform takes a simple form: he either proposes a single policy or constrains how far he can move policy in the direction of his bias.

**Lemma 1.** *In any equilibrium in which candidate  $i \in \{L, R\}$  wins with positive probability, he plays a pure strategy, choosing a platform  $A_i^*$  such that:*

1.  $A_R^*$  satisfies either (i)  $A_R^* = \{a_R^*\}$  with  $a_R^* \geq 0$ , or (ii)  $A_R^* = [\alpha_R(-1), \bar{a}_R^*]$  with  $\bar{a}_R^* \in [\bar{a}^0, \alpha_R(1)]$ ;
2.  $A_L^*$  satisfies either (i)  $A_L^* = \{a_L^*\}$  with  $a_L^* \leq 0$ , or (ii)  $A_L^* = [\underline{a}_L^*, \alpha_L(1)]$  with  $\underline{a}_L^* \in [\alpha_L(-1), \underline{a}^0]$ .

**Lemma 1** says that equilibrium platforms are singletons or intervals (for candidates who win with positive probability), analogous to voter-optimal platforms (**Proposition 1**). The key idea is that a candidate will only propose platforms that maximize a weighted average of his own utility and the voter's. Only singleton or interval platforms have this property; any other platform would neither help a candidate win the election, nor, conditional on winning, benefit him either.<sup>14</sup> A subtle but noteworthy feature of equilibrium strategies is that a candidate who wins with positive probability does not randomize over platforms.

<sup>14</sup> The formal proof of **Lemma 1** shows that one can map a winning candidate's program into an optimal

Notwithstanding the similarities, [Lemma 1](#) also points to an important distinction between equilibrium platforms and voter-optimal platforms. While both could be intervals that constrain a politician from choosing policies too far in the direction of his policy bias, the former may entail more—but not less—discretion. Our main result, [Proposition 3](#) develops this further by providing a full characterization of the equilibrium platforms. Recall that  $W_0(A, i)$  is the voter's expected utility when candidate  $i \in \{L, R\}$  is elected with platform  $A \subseteq \mathbb{R}$ .

**Proposition 3.** *Let  $b_R \leq -b_L$ , so that candidate  $R$  is no more biased than  $L$ .<sup>15</sup> An equilibrium exists. Furthermore:*

1. *If  $b_R \geq 1$ , then in any equilibrium  $A_i^* = A_i^0 = \{0\}$  for each  $i \in \{L, R\}$ .*
2. *If  $b_R = 0$ , then in any equilibrium if  $i \in \{L, R\}$  is elected he has  $b_i = 0$  and  $A_i^* = A_i^0 = [-1, 1]$ .*
3. *If  $b_R = -b_L \in (0, 1)$ , then in any equilibrium  $A_L^* = A_L^0 = [\underline{a}^0, \alpha_L(1)]$  and  $A_R^* = A_R^0 = [\alpha_R(-1), \bar{a}^0]$ .*
4. *If  $b_R \in (0, \min\{-b_L, 1\})$  and  $W_0(A_L^0, L) > W_0(\mathbb{R}, R)$ , then there is a unique equilibrium. It is a pure-strategy equilibrium in which (i)  $A_L^* = A_L^0$ , (ii)  $A_R^* = [\alpha_R(-1), \bar{a}_R^*]$ , where  $\bar{a}_R^* \in (\bar{a}^0, \alpha_R(1))$  is the unique solution to  $W_0(A_L^0, L) = W_0([\alpha_R(-1), \bar{a}_R^*], R)$ , and (iii) the voter elects  $R$ .*
5. *If  $b_R \in (0, \min\{-b_L, 1\})$  and  $W_0(A_L^0, L) \leq W_0(\mathbb{R}, R)$ , then there is a unique equilibrium outcome. In any equilibrium  $A_R^* = [\alpha_R(-1), \alpha_R(1)]$  and the voter elects  $R$ .*

[Proposition 3](#) shows that the winning platform is equal to the voter-optimal platform only in three special cases: (i) both candidates are extreme ([part 1](#)), (ii) at least one candidate is unbiased ([part 2](#)), or (iii) candidates' biases are perfectly symmetric ([part 3](#)). In any other case ([parts 4 and 5](#)), the more moderate candidate wins the election with certainty while obtaining more discretion than is voter-optimal. This does not imply, however, that the voter is better off with more extreme candidates because they propose their voter-optimal platform. In fact, as we establish later, more moderate candidates can only increase the voter's welfare because their policy choice is on average closer to the voter's preferred policy.

To understand [Proposition 3](#), begin by observing that when candidates  $L$  and  $R$  are biased but equally so ( $-b_L = b_R > 0$ ), they must each offer their voter-optimal platforms. If one

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delegation problem similar to the previous section's but with a fictitious principal whose utility is given by a suitably-chosen convex combination of the voter's and candidate's utilities. This problem is in turn isomorphic to the previous section's but with a scaled-down bias for the candidate.

<sup>15</sup> The case in which  $L$  is no more biased than  $R$ , i.e.  $b_R \geq -b_L$ , is analogous.

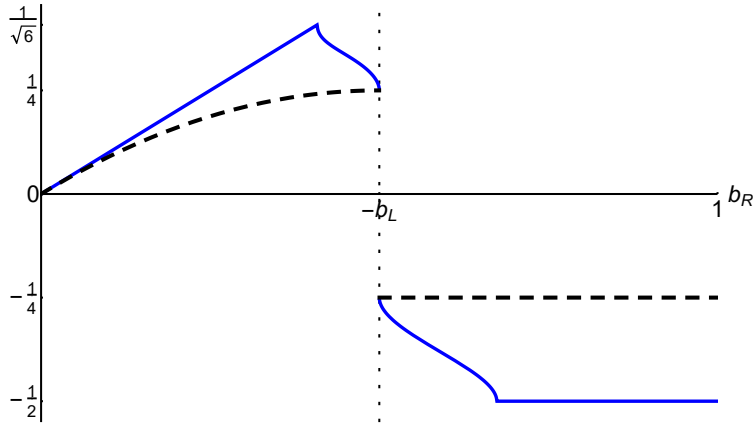
didn't, say  $L$  with  $A_L \neq A_L^0$ , then  $R$  can propose a platform  $A_R$  that (i) guarantees he wins the election (i.e.,  $A_R$  such that  $W_0(A_R, R) > W_0(A_L, L)$ ) and (ii) gives him more discretion than his voter-optimal platform (e.g.,  $A_R = [\alpha_R(-1), \bar{a}_R]$  with  $\bar{a}_R \in (\bar{a}^0, \alpha_R(1)]$ ). But such platforms cannot constitute an equilibrium because  $L$  would in turn prefer to offer a platform that provides the voter with higher utility than  $A_R$ , guaranteeing victory for  $L$  instead of  $R$ .<sup>16</sup>

Parts 4 and 5 of the Proposition follow from the fact that when candidate  $R$  is ex-ante advantaged because he is less biased than  $L$  ( $b_R < -b_L$ ) and not extreme ( $b_R < 1$ ), he can guarantee victory by offering his voter-optimal platform,  $A_R^0$ . But since he prefers more discretion, he will instead offer the most discretionary platform subject to the constraint of providing the voter at least as much utility as  $L$  does when  $L$  proposes his voter-optimal platform. The distinction between parts 4 and 5 concerns whether  $R$  could be defeated by  $L$  if  $R$  proposes a platform that is too discretionary. If  $L$ 's voter-optimal platform,  $A_L^0$ , is preferred by the voter to  $R$  having full discretion, then equilibrium forces  $L$  to offer precisely that platform and the voter to resolve her equilibrium indifference between the two candidates in favor of  $R$  (part 4). In this case we say that electoral competition is *effective*, since the presence of  $L$ , even though he doesn't win, influences the equilibrium outcome. If instead  $R$  is sufficiently advantaged because  $b_R$  is sufficiently smaller than  $-b_L$ , then  $R$  wins with full discretion no matter what platform  $L$  offers. In this case, any platform of  $L$  can be supported in equilibrium (part 5). Subsection 5.3 shows that adding aggregate uncertainty pins down both candidates' behavior regardless of their biases.

Figure 2 provides a graphical representation of the expected equilibrium policy, comparing it with the expected policy under optimal representation. When the bias of candidate  $R$  is small enough, he wins with full discretion, and so the equilibrium policy is always  $b_R$  higher than the voter's preferred policy; this explains the 45 degree line portion of the figure. As  $b_R$  further increases towards  $-b_L$ , candidate  $R$  is forced to place more constraints on his discretion in order to win, which decreases the expected policy. When  $b_R$  crosses the threshold  $-b_L$ ,  $L$  now wins the election, giving himself increasing amounts of discretion until  $b_R$  becomes large enough that  $L$  is elected with full discretion. Thereafter, increases in  $b_R$  have no effect on equilibrium policies, which explains the flat line portion of the figure. Notice that except when  $b_R = -b_L$  or when  $b_R = 0$ , the expected policy distortion in the direction of the winning candidate's bias is greater in equilibrium than under optimal representation.

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<sup>16</sup> It can be checked that there is such a platform that  $L$  would prefer to be elected with rather than letting  $R$  win with  $A_R$  (see Lemma A.4). Note that if  $b_R = b_L = 0$ , then it is not necessary that *both* candidates choose the voter-optimal platform  $[-1, 1]$ ; only one need do so because the voter could break indifference in favor of that candidate. This point explains the difference between parts 2 and 3 of Proposition 3.



**Figure 2** – Expected equilibrium policy as a function of  $b_R$ .

Candidate  $R$  is elected when  $b_R < -b_L$ ; candidate  $L$  is elected when  $b_R > -b_L$ . The blue solid curve is the expected equilibrium policy. The black dashed line is the expected policy under optimal representation. Parameter values:  $b_L = -1/2$ ,  $\theta \sim U[-1, 1]$ .

We now turn to the important implications of [Proposition 3](#). If electoral competition is not effective or if at least one candidate is unbiased, matters are straightforward: the advantaged candidate or one of the unbiased candidates gets elected and has full discretion. For this reason we focus in the rest of this section on effective electoral competition in which both candidates are biased.

First, our theory predicts that convergence does not generally occur.

**Implication 1.** *Convergence occurs if and only if both candidates are extreme.*

([Appendix A](#) details how this and subsequent implications follow from parts [1](#), [2](#), and [4](#) of [Proposition 3](#).)

Only when both candidates are extreme does the standard Downsian prediction hold in our model. In both settings, convergence is at the platform that maximizes a candidate's electoral chances; in our context, it is the voter's ex-ante optimal policy. The convergence in our setting entails a social cost of policy adaptability. Nevertheless, when it occurs, convergence is at the voter-optimal platform: the voter benefits from precluding policy adaptation when candidates are extreme.

When candidates' policy preferences are moderate (i.e., at least one candidate  $i$  has  $|b_i| < 1$ ), candidates propose divergent platforms. Further, divergence in platforms implies divergence in expected policies.

**Implication 2.** *Unless a candidate is extreme, the expected policy he chooses if elected is biased in the direction of his policy preference.*



As the voter's behavior is entirely predictable in our baseline model (which is relaxed in [Subsection 5.3](#)), the candidates' platforms are invariant to the office motivation parameter  $\phi$ . Hence, equilibrium divergence persists even as  $\phi \rightarrow \infty$ . By contrast, in canonical models of divergence with policy-motivated candidates ([Wittman, 1983](#); [Calvert, 1985](#)), divergence vanishes as candidates become primarily office motivated.

Divergence is intricately tied to ambiguity as only extreme candidates propose single policy platforms. More interesting is the relationship between ambiguity and electoral success. To state the next implication, we measure the extent of candidates' ambiguity by the *variance* of their ex-post policies.<sup>17</sup> A candidate's platform is ambiguous if and only if the variance of his ex-post policies is non-zero; a higher variance reflects more ambiguity in the sense of more uncertainty about the policy the candidate will choose if elected.

**Implication 3.** *When the two candidates are not equally ambiguous, the more ambiguous candidate always wins the election.*

This implication follows from part 4 of [Proposition 3](#). When candidate  $R$  is less biased than  $L$ , not only is  $R$ 's voter-optimal platform more ambiguous than  $L$ 's, but moreover,  $R$  can take advantage of his policy alignment by winning the election with a platform that has even greater ambiguity than his voter-optimal platform.  $L$  is unable to undercut  $R$  even though  $L$  offers his voter-optimal platform.

The electoral success of ambiguous candidates has long puzzled political scientists. Our theory shows that analysts must be careful with the direction of causality: in our model, a candidate does not win because he is more ambiguous than his opponent; rather, he is more ambiguous because he is able to win by exploiting his ex-ante electoral advantage of preference alignment.

Our next implication fleshes out a comparative static concerning policy divergence. We focus on the case with symmetric biases and consider how the degree of policy divergence depends on the level of political polarization. We measure the expected policy divergence by the difference in expected policies of the two candidates (i.e.,  $\mathbb{E}[a_R(\theta, A_R^*)] - \mathbb{E}[a_L(\theta, A_L^*)]$ ) and political polarization by the difference in policy preferences (i.e.,  $b_R - b_L$ ).

**Implication 4.** *When  $b_R = -b_L = b \in (0, 1)$ , expected policy divergence is non-monotonic in political polarization. Expected policy divergence is initially increasing in polarization (when  $b \approx 0$ ) and eventually decreasing in polarization (when  $b \approx 1$ ).*

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<sup>17</sup> We use the variance for simplicity. In equilibrium the candidate with a wider interval platform has greater variance in his ex-post policy ([Lemma A.9](#)).

Fixing any pair of platforms, at least one of which is ambiguous, an increase in polarization increases expected policy divergence. However, equilibrium platforms also change with polarization. In particular, symmetrically-biased candidates who become more polarized impose more stringent constraints on their policy choices in order to win the election. Consequently, expected policy divergence can decrease as polarization increases; it necessarily does when candidates become extreme, as platforms then converge to  $\{0\}$ . [Implication 4](#) has important consequences for the empirical study of political polarization. It demonstrates that any measure of average policy divergence is consistent with at least two levels of political polarization; higher political polarization is compatible with lower average policy divergence.

While electoral competition generally produces divergent and ambiguous platforms, it is beneficial to the voter because she values policy adaptability. This normative conclusion contrasts with most of the literature on electoral competition, which often views both divergence and ambiguity as undesirable.<sup>18</sup>

**Implication 5.** *The voter's equilibrium welfare in a divergent equilibrium is higher than it would be under convergence to singleton platforms.*

However, electoral competition in our framework does not generally lead to optimal outcomes for the voter.

**Implication 6.** *Effective electoral competition induces candidates to limit ambiguity in a manner desired by the voter. However, it is not sufficient to guarantee voter-optimal platforms unless candidates are symmetrically biased or extreme; in any other case, the winning candidate is overly ambiguous.*

Put differently, electoral competition benefits the voter as long as it is effective, but except in special cases, it does not lead to optimal political representation. Equilibrium political representation is generally a mixture of the trustee and delegate models, but overly tilted towards trusteeship from the voter's point of view. By contrast, in the traditional Downsian framework the (median) voter always obtains her preferred outcome.<sup>19</sup>

One salient case in which each candidate proposes his voter-optimal platform is when candidates are symmetrically biased.<sup>20</sup> Nonetheless, unless they are extreme they implement

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<sup>18</sup> For example, see [Fiorina, Abrams, and Pope \(2010\)](#) and [Azzimonti \(2011\)](#) on divergence and [Page \(1976\)](#) and [Aragones and Neeman \(2000\)](#) on ambiguity. Exceptions include [Bernhardt, Duggan, and Squintani \(2009\)](#) and [Van Weelden \(2013\)](#) on divergence and [Shepsle \(1972\)](#) and [Callander and Wilson \(2008\)](#) on ambiguity.

<sup>19</sup> As noted by [Bernhardt et al. \(2009\)](#), the Downsian model with policy motivated candidates and uncertainty about the median voter's location/preferences ([Wittman, 1983](#); [Calvert, 1985](#)) also entails inefficiency.

<sup>20</sup> Literally taken, this point owes to our assumption of a symmetric state distribution. The broader point is that voter-optimal platforms should only be expected when neither candidate has a pre-existing electoral advantage.

different policies when elected. As the voter is indifferent between the two candidates, she could toss a fair coin to determine the winner. It follows that *close elections*—elections that each candidate is equally likely to win—are compatible with divergent platforms and expected policy divergence. Our theory thus rationalizes Lee et al.’s (2004) empirical finding of policy divergence in close elections. In our framework, this does not imply that the electorate merely elects policies; rather, divergence in close elections is consistent with candidates proposing their voter-optimal platforms.

Finally, we highlight the disciplining effect of elections by noting that the voter’s welfare depends not only on the winning candidate’s bias, but also on the losing candidate’s.

**Implication 7.** *If the candidates are moderate and symmetrically biased, the voter’s utility is decreasing in the level of their bias. If the candidates are moderate and asymmetrically biased, the voter’s utility is decreasing in the losing candidate’s bias, and constant in the winning candidate’s bias.*

When the candidate’s are symmetrically biased, candidates propose their voter-optimal platform in equilibrium, and voter welfare is decreasing in the candidates’ bias. In an effective election with asymmetric candidates the more moderate candidate wins while making the voter indifferent between the candidates; hence, the voter’s utility is determined by the utility she would receive from the voter-optimal platform of the losing candidate. It should be noted that [Implication 7](#) refers only to effective elections: when the candidates’ biases are sufficiently asymmetric, the more moderate candidate wins the election while proposing full discretion, and the voter’s utility depends only on the bias of the winning candidate.

## 5. Extensions

We now discuss the robustness of our main results to some extensions.

### 5.1. Heterogenous Voters

Suppose there are  $2N + 1$  ( $N \in \mathbb{N}$ ) voters. Each voter  $v \in \{v_{-N}, \dots, v_{-1}, 0, v_1, \dots, v_N\}$  has utility function

$$u_v(a, \theta) := -(a - v - \theta)^2,$$

with  $v_{-N} < \dots < v_{-1} < 0 < v_1 < \dots < v_N$ . The median voter,  $v = 0$ , has the same utility function as the representative voter in our baseline model. Owing to quadratic loss utility functions, the median voter is decisive.

**Proposition 4.** *Consider any pair of platforms,  $A_i$  and  $A_j$ . Electing candidate  $i \in \{L, R\}$  with platform  $A_i$  is preferred to electing candidate  $j \in \{L, R\}$  with platform  $A_j$  by a majority of voters if and only if  $W_0(A_i, i) > W_0(A_j, j)$ .*

Proposition 4 implies that the optimal platform for candidate  $i$  from the perspective of the median voter is the unique Condorcet winner among  $i$ 's platforms in a heterogeneous electorate. It follows that the equilibrium characterization in Proposition 3 and all our other insights remain valid with a heterogeneous electorate.

## 5.2. Alternative Assumptions About Commitment

We have assumed that candidates' campaign promises take the form of a set of policy options. We now discuss why this assumption is key for our results, returning to the single-voter model for simplicity.

Suppose first that candidates are constrained to offer the voter only a single policy. The voter's ex-ante preferred policy is 0, and candidates face the same strategic problem as in the classical Downsian framework. Equilibrium platforms are non-ambiguous (by stipulation) and convergent (in equilibrium). Precluding ambiguous platforms necessarily reduces the voter's welfare unless both candidates are extreme, in which case it has no effect.

Suppose next that candidates can make state-contingent promises. For example, they might announce the level of benefit cuts for each possible contingency, effectively making a pronouncement such as "If the budgetary deficit turns out to be  $\theta_1$  then I will cut benefits by  $a_1$ , if it is  $\theta_2$  then I will cut benefits by  $a_2$ , ..." Such commitments are enforceable only if the state of the world is easily observable by the voter, which it often is not. But if such commitments were feasible, there would again be convergence: both candidates would commit to implementing a policy that matches the state of the world. Non-verifiability of the state may thus harm the voter in two ways: not only could the office-holder tilt policies towards his own bliss point given his platform, but the winning platform itself may be over-ambiguous.

## 5.3. Probabilistic Voting

We have assumed that elections are deterministic in the sense that candidates can perfectly predict how the electorate will respond to their platforms. Suppose now that after candidates choose their platforms, the voter's preference is affected by a "valence" shock before she votes. Denoting the valence shock by  $\eta$  and the elected candidate by  $e$ , we suppose the voter's utility function is  $u_0(a, \theta, e) := -(a - \theta)^2 + \eta \mathbb{I}_{\{e=R\}}$ . Thus  $\eta > 0$  ( $\eta < 0$ ) corresponds to candidate  $R$  ( $L$ ) having a valence advantage.

It is common knowledge that  $\eta$  is distributed on the interval  $[-\lambda, \lambda]$  according to a CDF  $G$  with a positive and differentiable density  $g$ . To simplify the exposition, we focus on candidates who are biased but not extreme, and we assume that the valence shock can be “large enough”: even a candidate with  $|b_i| = 1$  proposing full discretion wins with positive probability against an unbiased candidate proposing full discretion.<sup>21</sup> When candidate  $L$  proposes platform  $A_L$  and candidate  $R$  proposes platform  $A_R$ , the voter now elects  $R$  (ignoring indifference, which will be an event with zero probability) if and only if  $W_0(A_L, L) < W_0(A_R, R) + \eta$ .

It is well known since [Wittman \(1983\)](#) and [Calvert \(1985\)](#) that uncertainty about the voter’s preferences induces policy-motivated candidates to propose divergent platforms. Intuitively, probabilistic voting reduces a candidate’s electoral cost of tilting platforms towards his own preferred platform. This force is also in effect in our setting.

**Proposition 5.** *Suppose  $|b_i| \in (0, 1)$  for each  $i \in \{L, R\}$ . Then in every pure-strategy equilibrium:*

1.  $A_L^* = [\underline{a}_L^*, \alpha_L(1)]$  with  $\underline{a}_L^* \in (\alpha_L(-1), \underline{a}^0)$ , and  $A_R^* = [\alpha_R(-1), \bar{a}_R^*]$  with  $\bar{a}_R^* \in (\bar{a}^0, \alpha_R(1))$ .
2. For  $i \in \{L, R\}$ ,  $\lim_{\phi \rightarrow \infty} A_i^* = A_i^0$ .

[Proposition 5](#) characterizes candidates’ behavior in any pure-strategy equilibrium.<sup>22</sup> Under probabilistic voting, even symmetric candidates propose more ambiguous platforms than is voter-optimal ([Proposition 5](#) part 1), in contrast to our baseline result ([Proposition 3](#) part 3). The reason is that the electoral cost of proposing a slightly more ambiguous platform is second order, whereas the benefit from having more discretion when elected is first order. The degree of divergence from the voter-optimal platform turns on how much candidates care about office relative to policy, as in [Wittman \(1983\)](#) and [Calvert \(1985\)](#). As office motivation becomes paramount, both candidates try to maximize their probability of winning, which leads them towards proposing their respective voter-optimal platforms even when they are asymmetrically biased ([Proposition 5](#) part 2). Thus, probabilistic voting can either increase or decrease ambiguity, depending on candidates’ biases and office motivation.

Despite the new features induced by probabilistic voting, one central point remains unchanged. When neither candidate has an ex-ante valence advantage (i.e.,  $G(0) = 1/2$ ), greater ambiguity is associated with greater electoral success, at least when candidates are sufficiently office motivated. As before, this positive correlation is a consequence of a more moderate candidate’s ex-ante electoral advantage.

<sup>21</sup> Formally, we assume  $\lambda \geq -W_0([\alpha_i(-1), \alpha_i(1)], i)$  for all  $b_i \in [-1, 1]$ .

<sup>22</sup> Existence of pure-strategy equilibria in probabilistic voting models requires conditions on the distribution of aggregate uncertainty. [Appendix B](#) provides a condition for our setting; it also assures the existence of mixed-strategy equilibria and shows that versions of [Proposition 5](#) and [Proposition 6](#) hold for all equilibria.

**Proposition 6.** *Assume  $|b_i| < |b_{-i}|$  for some  $i \in \{L, R\}$  and focus on pure strategy equilibria. If  $G(0) = 1/2$ , then there exists a  $\bar{\phi}$  such that when  $\phi > \bar{\phi}$ , candidate  $i$  is more ambiguous than his opponent and wins with probability greater than  $1/2$ .*

## 6. Conclusion

This paper introduces a new model of electoral competition in which policy relevant information is revealed to the elected politician after he takes office. Our framework is well suited to formally studying a classical issue in the theory of political representation. We find that optimal political representation consists of a mixture of the trustee and delegate models: voters would like politicians to have some discretion to adapt policies to changing circumstances, but also constrain how much they can do so to mitigate policy bias. Discretion is obtained through ambiguous electoral platforms. While electoral competition generally benefits voters, it need not lead to optimal political representation; there is instead a slant towards excess trusteeship. Candidates typically propose divergent and ambiguous platforms. Our theory thus provides a single rationale for two well-documented empirical facts. We also find that the more ambiguous candidate is more likely to win. However, ambiguity does not cause electoral success; rather, the correlation is a consequence of a more moderate candidate's pre-existing electoral advantage.

To isolate the channel underlying our results, we have assumed that candidates' policy biases are known. Voter uncertainty about these biases introduces additional issues, in particular whether a candidate's platform can serve as a signal of his bias. We can show that, under some conditions, there are pooling equilibria that preserve our main insights and that the scope for separation is in fact limited (cf. [Tanner, 2013](#)). A thorough analysis is left for future research.

We have modeled an election's outcome as entirely determined by the median voter. In practice, candidates may target other constituencies: voters in the party base (both because of primaries and within general elections), interest groups, donors, etc. Our logic suggests that if electoral pressures lead a candidate to target a constituency more extreme than himself, he may make commitments that limit his ability to act contrary to his bias (where extremism and bias are relative to the median voter). For example, at the behest of Grover Nordquist's Americans for Tax Reform, many Republicans have pledged to never vote for a tax increase. It would be fruitful to study how electoral competition shapes political representation with multiple constituencies.

Our results rely on the (common) assumption that candidates can credibly commit to their

platforms. The degree of commitment is limited, however, as candidates cannot commit to which policy they will implement from an ambiguous platform. In practice, candidates do sometimes break campaign pledges. Another interesting avenue for future research would be to extend our framework to allow for campaign promises to be broken at a cost.



# Appendices

## A. Proofs

### A.1. State Distributions Satisfying our Assumptions

We show here that our technical assumptions on the distribution of states are satisfied by familiar families of distributions.

**Remark 1.** Both (2) and (3) hold for (i) the uniform distribution on  $[-1, 1]$  and (ii) the truncated normal distribution on  $[-1, 1]$  with mean zero and variance of at least one.

**Proof.** Uniform and truncated normal distributions have log concave densities (Bagnoli and Bergstrom, 2005, Table 1 and Theorem 9), which is a sufficient condition for (3) to hold (Bagnoli and Bergstrom, 2005, Theorems 5 and 6). It is trivial that (2) holds for the uniform distribution, as the density is constant. Finally, it is straightforward to compute that for the truncated normal distribution with mean zero,  $\sigma^2 f'(\theta) = \theta f(\theta)$ , which implies (2) when  $\sigma^2 \geq 1$ .  $\square$

**Remark 2.** Given any distribution that has a log-concave positive density that is twice differentiable and symmetric around its mean, a suitable truncation and rescaling results in a density on  $[-1, 1]$  that satisfies (2) and (3).

**Proof.** Fix any log-concave twice differentiable density  $h(\theta) > 0$  on  $[a, b]$  with  $-\infty \leq a < b \leq \infty$ , and denote  $m := (a + b)/2$  if  $a$  and  $b$  are finite and  $m := 0$  otherwise (note that by symmetry,  $a$  is finite if and only if  $b$  is). By symmetry,  $h'(m) = 0$ ; further,  $-1 \leq h'/h \leq 1$  in some neighborhood of  $m$ . We proceed in two steps.

First, if needed, truncate  $h$  to an interval symmetric around  $m$  of length no more than two in which  $-1 \leq h'/h \leq 1$  throughout. With an abuse of notation, let the new density again be denoted by  $h$  and the new endpoints by  $a$  and  $b$ . Log-concavity of the density is preserved (Bagnoli and Bergstrom, 2005, Theorem 9), and we now have  $-1 \leq h'/h \leq 1$  on  $[a, b]$ .

Second, define a density on  $[-1, 1]$  by  $f(\theta) = h((\theta + 1)(b - a)/2 + a)/M$ , where  $M$  is a constant determined by the requirement  $\int_{-1}^1 f(\theta) d\theta = 1$ . Since  $(\theta + 1)(b - a)/2 + a$  is a linear function of  $\theta$ ,  $f(\theta)$  is again log-concave (Bagnoli and Bergstrom, 2005, Theorem 7), and hence (3) holds (Bagnoli and Bergstrom, 2005, Theorems 5 and 6). Since  $\frac{f'(\theta)}{f(\theta)} = \frac{h'((\theta+1)(b-a)/2+a)}{h((\theta+1)(b-a)/2+a)} \left(\frac{b-a}{2}\right)$ , and  $b - a \in (0, 2]$ , it follows that (2) is also satisfied.  $\square$

## A.2. Proofs for Section 3

The proof of [Proposition 1](#) uses [Alonso and Matouschek's \(2008\)](#) characterization of optimal delegation sets. To invoke their results, we follow [Alonso and Matouschek \(2008, p. 264\)](#) and define for each  $i \in \{L, R\}$  the *backwards bias* function

$$T_i(\theta) := F(\theta)\alpha_i(\theta) - \int_{-1}^{\theta} z f(z) dz, \quad (5)$$

and the *forward bias* function

$$S_i(\theta) := (1 - F(\theta))\alpha_i(\theta) - \int_{\theta}^1 z f(z) dz. \quad (6)$$

**Lemma A.1.**  $T_i(\theta)$  is convex when  $b_i \in (-1, 1)$ .

**Proof.** Twice differentiating  $T_i(\theta)$  and recalling that  $\alpha_i(\theta) = b_i + \theta$  yields  $T_i''(\theta) = b_i f'(\theta) + f(\theta)$ . The result follows from (2).  $\square$

**Proof of Proposition 1.** We only provide the proof for candidate  $R$ ; owing to the symmetry of the state density,  $f(\cdot)$ , the arguments are symmetric for candidate  $L$ .

Case 1: Suppose  $b_R \geq 1$ . Since  $S_R(t) = (1 - F(t))(\alpha_R(t) - \mathbb{E}[\theta|\theta \geq t])$ , the sign of  $S_R(t)$  is the same as that of  $b_R + t - \mathbb{E}[\theta|\theta \geq t]$ . This expression is increasing in  $t$  by (3) and consequently has range  $[b_R - 1, b_R] \subset \mathbb{R}_+$ . Thus,  $S_R(\theta) \geq 0$  for all  $\theta \in [-1, 1]$ . By [Alonso and Matouschek \(2008, Proposition 1\)](#), the voter-optimal platform entails no discretion, which implies  $A_R^0 = \{0\}$ , i.e. the singleton consisting of the ex-ante optimal policy.

Case 2: Suppose  $b_R \in [0, 1)$ . Define  $\bar{\theta}_R$  as the solution to

$$b_R + \bar{\theta}_R = \mathbb{E}[\theta|\theta \geq \bar{\theta}_R]. \quad (7)$$

Note that  $\bar{\theta}_R \in (-1, 1]$  is unique because  $t - \mathbb{E}[\theta|\theta \geq t]$  is increasing by (3). Using the same logic as in the previous case, it holds that  $\text{sign}[S_R(\theta)] = \text{sign}[\theta - \bar{\theta}_R]$ .

Moreover, since  $T_R(t) = F(t)(\alpha_R(t) - \mathbb{E}[\theta|\theta \leq t])$  and  $T_R(-1) = 0$ , condition (3) implies that  $T_R(\cdot)$  is increasing and hence always non-negative. Together with [Lemma A.1](#), these facts about  $S_R(\cdot)$  and  $T_R(\cdot)$  show that the conditions in [Alonso and Matouschek \(2008, Proposition 6\)](#) for “upper threshold” delegation to be optimal are satisfied. Thus, those authors’ arguments yield that the voter-optimal platform is the interval  $A_R^0 = [\alpha_R(-1), \alpha_R(\bar{\theta}_R)]$ . Comparing

Equation 7 and the definition of  $\bar{a}^0$  in Equation 4, it follows that  $\bar{a}^0 = b_R + \bar{\theta}_R = \alpha_R(\bar{\theta}_R) \in (0, 1)$ , and hence  $A_R^0 = [\alpha_R(-1), \bar{a}^0]$ .  $\square$

**Proof of Proposition 2.** We provides proofs for  $i = R$ ; the arguments are analogous for  $i = L$ . Assume  $b_R \in (0, 1)$ . We use the fact that  $\bar{a}^0 = b_R + \bar{\theta}_R$ , where  $\bar{\theta}_R$  is defined by Equation 7.

Part 1: Condition (3) and Equation 7 imply that  $\bar{\theta}_R$  is decreasing in  $b_R$ ; hence the right-hand side (RHS) of Equation 7, which is equal to  $\bar{a}^0$ , is also decreasing in  $b_R$ .

Part 2: Observe that

$$a_R(\theta, A_R^0) = \begin{cases} b_R + \theta & \text{if } \theta \leq \bar{\theta}_R, \\ b_R + \bar{\theta}_R & \text{otherwise.} \end{cases} \quad (8)$$

Thus,  $W_0(A_R^0, R) = -b_R^2 F(\bar{\theta}_R) - \int_{\bar{\theta}_R}^1 (b_R + \bar{\theta}_R - \theta)^2 f(\theta) d\theta$ . The envelope theorem and Equation 7 imply that  $\frac{dW_0(A_R^0, R)}{db_R} = -2b_R F(\bar{\theta}_R) < 0$ , where the inequality owes to  $b_R > 0$  and  $\bar{\theta}_R > -1$ .

Part 3: Since  $\mathbb{E}[b_R + \theta] = b_R$ , it follows from Equation 8 that

$$\mathbb{E}[a_R(\theta, A_R^0)] = b_R - (1 - F(\bar{\theta}_R)) \mathbb{E}[b_R + \theta - (b_R + \bar{\theta}_R) | \theta \geq \bar{\theta}_R] = F(\bar{\theta}_R) b_R,$$

where the second equality uses Equation 7. The desired results follow from the above simplification by noting that  $b_R > 0$ ,  $\bar{\theta}_R > -1$ , and  $\lim_{b_R \rightarrow 1} \bar{\theta}_R = -1$ .  $\square$

### A.3. Proof of Lemma 1

To aid with subsequent proofs, we will prove a result (Proposition A.1 below) that is stronger than Lemma 1. To state it, let  $\mathcal{A}_i^{PO}$  be the set of platforms that constitute a pairwise Pareto frontier between a candidate  $i \in \{L, R\}$  and the voter. More precisely, for each  $i \in \{L, R\}$ , define  $\mathcal{A}_i^{PO}$  as the set of  $i$ 's platforms such that for each  $A_i \in \mathcal{A}_i^{PO}$ , there exists a  $\delta \in [0, 1]$  such that  $A_i$  maximizes

$$\delta W_i(A_i, i) + (1 - \delta) W_0(A_i, i). \quad (9)$$

**Proposition A.1.** *In any equilibrium in which candidate  $i \in \{L, R\}$  wins with positive probability, he plays a pure strategy of choosing a platform in  $\mathcal{A}_i^{PO}$ . Moreover,*

$$\mathcal{A}_L^{PO} = \begin{cases} \{[\underline{a}, \alpha_L(1)] : \underline{a} \in [\alpha_L(-1), \underline{a}^0]\} & \text{if } b_L > -1, \\ \{\{a\} : a \in [\alpha_L(1), 0]\} \cup \{[\underline{a}, \alpha_L(1)] : \underline{a} \in [\alpha_L(-1), \alpha_L(1))\} & \text{if } b_L \leq -1, \end{cases} \quad (10)$$

$$\mathcal{A}_R^{PO} = \begin{cases} \{[\alpha_R(-1), \bar{a}] : \bar{a} \in [\bar{a}^0, \alpha_R(1)]\} & \text{if } b_R < 1, \\ \{\{a\} : a \in [0, \alpha_R(-1)]\} \cup \{[\alpha_R(-1), \bar{a}] : \bar{a} \in (\alpha_R(-1), \alpha_R(1)]\} & \text{if } b_R \geq 1. \end{cases} \quad (11)$$

The proof of [Proposition A.1](#) consists of a number of steps. We begin with [Lemma A.2](#), which shows why the characterizations in [Equation 10](#) and [Equation 11](#) are true. Given any platform  $A$  and  $x \in \mathbb{R}$ , we write  $A + x := \{a + x : a \in A\}$ .

**Lemma A.2.** *For any  $i \in \{L, R\}$  and any weight  $\delta \in [0, 1]$ , there is a unique  $A_i^\delta$  that maximizes (9).  $A_i^\delta = \tilde{A}_i^\delta + \delta b_i$ , where  $\tilde{A}_i^\delta$  is the voter-optimal platform when candidate  $i$ 's bias is  $(1 - \delta)b_i$ . Consequently, by [Proposition 1](#), (10) and (11) hold.*

**Proof.** For any weight  $\delta \in [0, 1]$ ,  $A_i^\delta$  maximizes expression (9) if and only if it is a solution to an optimal delegation problem where the candidate's utility function continues to be  $-(a - b_i - \theta)^2$  but the voter's utility function is instead

$$-\delta(a - \theta - b_i)^2 - (1 - \delta)(a - \theta)^2 = -(a - \delta b_i - \theta)^2 - \delta(1 - \delta)b_i^2.$$

We can just as well ignore the constant  $-\delta(1 - \delta)b_i^2$  and take the voter's utility function to be  $-(a - \delta b_i - \theta)^2$ . With a change of variables  $\tilde{a} = a - \delta b_i$ , we see that this problem is isomorphic to our baseline problem when the candidate's bias is  $(1 - \delta)b_i$ ; i.e., to a problem where the voter's utility function is  $-(\tilde{a} - \theta)^2$  and the candidate's is  $-(\tilde{a} - (1 - \delta)b_i - \theta)^2$ . Given the solution  $\tilde{A}_i^\delta$  to that problem, reversing the change of variables yields the solution  $A_i^\delta = \tilde{A}_i^\delta + \delta b_i$ .

[Equation 10](#) and [Equation 11](#) follow from the above characterization and the characterization of voter-optimal platforms in [Proposition 1](#).  $\square$

Our goal now is to show that any candidate  $i \in \{L, R\}$  who wins with positive probability must play a pure strategy and choose a platform in  $\mathcal{A}_i^{PO}$ . This is the content of [Lemma A.6](#) and [Lemma A.7](#) below. We proceed via three intermediate lemmas. Throughout, a candidate's mixed strategy is viewed as a probability measure on the (Borel-)measurable sets of platforms.

**Lemma A.3.** *Fix a candidate  $i \in \{L, R\}$  and any strategy for  $-i$ . Any best response for  $i$  must put zero probability on the set of platforms  $\mathcal{A}_i = \{A_i : A_i \notin \mathcal{A}_i^{PO} \text{ and } A_i \text{ wins with positive probability}\}$ .*

Roughly, the lemma says that against any strategy for candidate  $-i$ , any best response for  $i$  only uses platforms that either lose for sure or maximize a weighted average of  $i$ 's and the voter's utilities.

**Proof.** Suppose not, per contra. First observe that  $i$ 's strategy must put 0 probability on any subset of  $\mathcal{A}_i$  such that  $i$  would prefer to lose than win no matter his realized platform from the subset; otherwise,  $i$  can increase his payoff by shifting all mass from the subset to his full-discretion platform,  $[\alpha_i(-1), \alpha_i(1)]$ , a contradiction. Since  $\mathcal{A}_i$  is disjoint from  $\mathcal{A}_i^{PO}$ , the characterization of  $\mathcal{A}_i^{PO}$  in Equation 10 and Equation 11 implies that for each  $A_i \in \mathcal{A}_i$  there exists a platform  $\tilde{A}_i \in \mathcal{A}_i^{PO}$  such that  $W_0(\tilde{A}_i, i) \geq W_0(A_i, i)$  and  $W_i(\tilde{A}_i, i) > W_i(A_i, i)$ . By changing each platform  $A_i \in \mathcal{A}_i$  to  $\tilde{A}_i$ ,  $i$  would not decrease his probability of winning while increasing his expected utility conditional on winning, contradicting that he is playing a best response.  $\square$

**Lemma A.4.** Fix any platforms  $A_L$  and  $A_R$  with  $\mathbb{E}[a_R(\theta, A_R)] \geq \mathbb{E}[a_L(\theta, A_L)]$ . It holds for each  $i \in \{L, R\}$  that

$$W_0(A_i, i) \geq W_0(A_{-i}, -i) \implies W_i(A_i, i) \geq W_i(A_{-i}, -i) + \phi.$$

Thus, if  $A_i$  wins with positive probability against  $A_{-i}$ ,  $i$  prefers winning with  $A_i$  to losing against  $A_{-i}$ .

**Proof.** Given any platforms  $A_L$  and  $A_R$ , a routine computation establishes that for  $i \in \{L, R\}$ ,

$$\begin{aligned} W_i(A_i, i) - W_i(A_{-i}, -i) &= \phi - \int_{-1}^1 [(a_i(\theta, A_i) - \theta)^2 - (a_{-i}(\theta, A_{-i}) - \theta)^2] f(\theta) d\theta \\ &\quad + \int_{-1}^1 2b_i [(a_i(\theta, A_i) - a_{-i}(\theta, A_{-i}))] f(\theta) d\theta, \end{aligned}$$

and

$$W_0(A_i, i) - W_0(A_{-i}, -i) = - \int_{-1}^1 [(a_i(\theta, A_i) - \theta)^2 - (a_{-i}(\theta, A_{-i}) - \theta)^2] f(\theta) d\theta.$$

Hence,

$$W_i(A_i, i) - W_i(A_{-i}, -i) - [W_0(A_i, i) - W_0(A_{-i}, -i)] = \phi + 2b_i \{\mathbb{E}[a_i(\theta, A_i)] - \mathbb{E}[a_{-i}(\theta, A_{-i})]\}.$$

The result follows because, by hypothesis,  $2b_i \{\mathbb{E}[a_i(\theta, A_i)] - \mathbb{E}[a_{-i}(\theta, A_{-i})]\} \geq 0$ .  $\square$

**Lemma A.5.** Fix  $i \in \{L, R\}$  and some strategy for candidate  $-i$  that puts zero probability on

$$\{A_{-i} : b_{-i} \mathbb{E}[a_{-i}(\theta, A_{-i})] < 0 \text{ and } W_0(A_{-i}, -i) \leq W_0(A_i^0, i)\}.$$

Assume that playing  $A_i^0$  gives  $i$  a positive probability of winning. Then, any best response for  $i$  puts

zero probability on the set of platforms with which  $i$  would lose with probability one.

Roughly, the lemma says that if  $-i$  only uses platforms that would result in expected policies opposite to  $i$ 's bias whenever they tie or win against  $i$ 's voter-optimal platform, then either  $i$  will lose for sure no matter the platform he proposes, or his best response cannot use any platform that loses for sure. In particular, his best response cannot mix over platforms that lose for sure and platforms that don't.

**Proof.** Suppose not, per contra. Consider an alternative strategy that is identical to the given best response except for shifting all probability mass away from  $A_i$  to the platform  $A_i^0$ . The difference in  $i$ 's payoff from playing the new strategy versus the given best response is proportional to

$$\mathbb{E} [\Pr(A_i^0 \text{ wins against } A_{-i}) (W_i(A_i^0, i) - W_i(A_{-i}, -i))] , \quad (12)$$

where the expectation is taken using both  $-i$ 's strategy and the voter's behavior (specifically, her behavior when indifferent). As  $\Pr(A_i^0 \text{ wins against } A_{-i}) > 0$  only if  $W_0(A_i^0, i) \geq W_0(A_{-i}, -i)$  and by hypothesis there is positive probability on  $\{A_{-i} : A_i^0 \text{ wins against } A_{-i}\}$ , Lemma A.4 implies that expression (12) is positive. It follows that the alternative strategy is a better response than the given best response, a contradiction.  $\square$

**Lemma A.6.** *In any equilibrium, if  $i \in \{L, R\}$  wins with positive probability, then his strategy puts probability one within the set of platforms  $A_i^{PO}$ .*

**Proof.** Fix  $i \in \{L, R\}$  and an equilibrium in which  $i$  wins with positive probability. Lemma A.3 implies that with probability one, whenever  $i$  is elected his platform is in  $A_i^{PO}$ . Any such platform  $A_i$  has  $b_i \mathbb{E}[a_i(\theta, A_i)] \geq 0$  because of Proposition 2 part 3. If  $i$  wins with probability one, we are done, so suppose  $-i$  wins with positive probability as well. As the above points now hold analogously for  $-i$ , whenever  $-i$  is elected his platform has  $b_{-i} \mathbb{E}[a_{-i}(\theta, A_{-i})] \geq 0$ . We can then apply Lemma A.5 to conclude that since  $i$  is playing a best response, he puts zero probability on any set of platforms that all lose with probability one. It follows from Lemma A.3 that with probability one  $i$  must play a platform in  $A_i^{PO}$ .  $\square$

**Lemma A.7.** *In any equilibrium, a candidate who wins with positive probability plays a pure strategy. Moreover, if both win with positive probability, each candidate  $i \in \{L, R\}$  chooses  $A_i^0$ .*

**Proof.** If one candidate  $i$  wins with probability one, then  $W_i(A_i, i)$  must be constant across all platforms in the support of his strategy. Since this support is contained in  $A_i^{PO}$  (Lemma A.6) and  $W_i(A_i^\delta, i)$  is increasing in  $\delta$ ,  $i$  must be playing a pure strategy.<sup>23</sup>

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<sup>23</sup> Recall  $A_i^\delta = \tilde{A}_i^\delta + \delta b_i$ , where  $\tilde{A}_i^\delta$  is the voter-optimal platform when candidate  $i$ 's bias is  $(1 - \delta)b_i$ .

So assume both candidates win with positive probability. By [Lemma A.6](#), we can view each candidate  $i$ 's strategy as CDF  $H_i(\delta)$  over weights  $\delta \in [0, 1]$ , where the platform corresponding to  $\delta$  is  $A_i^\delta$  defined in [Lemma A.2](#). For each candidate  $i$ , let  $\bar{\delta}_i := \sup\{\delta : H_i(\delta) < 1\}$  be the supremum of  $i$ 's support. If  $\bar{\delta}_L = \bar{\delta}_R = 0$ , both candidates are playing a pure strategy. So, suppose to contradiction that  $\max\{\bar{\delta}_L, \bar{\delta}_R\} > 0$ . Conditional on a candidate being elected, the voter's (resp., the candidate's) utility is continuous and decreasing (resp., increasing) in that candidate's choice of  $\delta$ . We make two claims:

1. The voter must be indifferent over the two candidates when they each choose  $\bar{\delta}_i$ : otherwise, if  $W_0(A_{-i}^{\bar{\delta}_i}, -i) > W_0(A_i^{\bar{\delta}_i}, i)$  for some  $i$ , then  $i$  loses with probability one when he chooses any  $\delta$  in a neighborhood containing  $\bar{\delta}_i$ , contradicting [Lemma A.5](#).
2.  $\lim_{\delta \uparrow \bar{\delta}_i} H_i(\delta) = 1$ , i.e. there is no atom on  $\bar{\delta}_i$ . Suppose there were: then  $-i$  cannot have an atom on  $\bar{\delta}_i$  because each candidate  $j \in \{L, R\}$  would then prefer a slightly lower weight than  $\bar{\delta}_j$  (using the previous claim and [Lemma A.4](#)), and it is feasible for at least one candidate to choose a lower weight (by the assumption that  $\max\{\bar{\delta}_L, \bar{\delta}_R\} > 0$ ). But  $i$  being the only candidate with an atom at  $\bar{\delta}_i$  contradicts [Lemma A.5](#) because  $\bar{\delta}_i$  is played by  $i$  with positive probability despite winning with zero probability.

Therefore, each candidate  $i$  wins with probability (arbitrarily close to) zero when he chooses  $(\delta \text{ arbitrarily close to}) \bar{\delta}_i$ , which implies that the expected payoff for  $i$  from choosing a platform (arbitrarily close to)  $\bar{\delta}_i$  is (arbitrarily close to)  $\mathbb{E}[W_i(A_{-i}^{\bar{\delta}_i}, -i)]$ . On the other hand,  $i$ 's expected payoff from choosing any platform that wins with positive probability—and such platforms exist as both candidates win with positive probability—is larger because of [Lemma A.4](#), given that  $-i$ 's strategy's support is within  $\mathcal{A}_{-i}^{PO}$ . Thus,  $i$  is not indifferent over all platforms in his strategy's support, a contradiction.

When both candidates win with positive probability, the above argument establishing that  $\max\{\bar{\delta}_L, \bar{\delta}_R\} = 0$  implies that both candidates are proposing their respective voter-optimal platforms,  $A_L^0$  and  $A_R^0$ , by the definition of  $\bar{\delta}_L$  and  $\bar{\delta}_R$ .  $\square$

[Proposition A.1](#) (and hence [Lemma 1](#)) follows from [Lemma A.7](#), [Lemma A.6](#), and [Lemma A.2](#).

#### A.4. Proof of [Proposition 3](#)

Throughout this proof, let  $b_R \leq -b_L$ . We will repeatedly use the following facts. By [Proposition A.1](#), a candidate  $i$  who wins with positive probability plays a pure strategy of choosing a platform  $A_i^\delta$  for some  $\delta \in [0, 1]$ , where  $A_i^\delta$  was characterized in [Lemma A.2](#). We



will sometimes refer to such a candidate as simply choosing  $\delta \in [0, 1]$ . An implication of [Proposition A.1](#) and [Lemma A.5](#) is that if candidate  $i$  wins with positive probability, then  $-i$  prefers to win (rather than lose) with any platform that can win. Lastly, for each  $i \in \{L, R\}$ ,  $W_i(A_i^\delta, i)$  (resp.,  $W_0(A_i^\delta, i)$ ) is continuous and increasing (resp., decreasing) in  $\delta$ .

The following lemma simplifies the analysis substantially.

**Lemma A.8.** *There is an equilibrium in which both candidates win with positive probability if and only if either  $b_L = -b_R$  or both candidates are extreme.*

**Proof.** If both candidates win with positive probability, each candidate  $i \in \{L, R\}$  chooses  $A_i^0$  ([Lemma A.7](#)). If  $b_R < \min\{-b_L, 1\}$  then  $W_0(A_R^0, R) > W_0(A_L^0, L)$  by [item 2 of Proposition 2](#), so there is no equilibrium in which both win with positive probability. Conversely, if  $b_R = -b_L$  or both candidates are extreme, then  $W_0(A_R^0, R) = W_0(A_L^0, L)$  and such an equilibrium exists.  $\square$

We now proceed with the proof of [Proposition 3](#).

[Parts 1 and 3](#): Existence follows from [Lemma A.8](#). To see that all equilibria have platforms  $A_L^0$  and  $A_R^0$ , note that there is no equilibrium in which a candidate  $i$  wins with probability one and proposes  $A_i \neq A_i^0$ ; for, the opponent  $-i$  could propose  $A_{-i} = A_{-i}^0$  and instead win. The only other possibility is that both candidates win with positive probability, in which case the results follow from [Lemma A.7](#) and [Lemma A.8](#).

[Part 2](#): Let  $i \in \{L, R\}$  be a candidate with  $b_i = 0$ . It is an equilibrium for  $i$  to propose  $A_i^* = A_i^0 = [-1, 1]$  and the voter to elect  $i$  regardless of  $-i$ 's platform, no matter  $-i$ 's strategy. By [Proposition A.1](#), in any equilibrium if  $i$  is elected with positive probability his platform must be  $A_i^*$ . Suppose there is an equilibrium in which  $-i$  is elected with positive probability. Since  $i$  prefers to be elected with any platform, he must be playing platform  $A_i^*$  and it further holds that  $b_{-i} = 0$  and  $-i$  must be playing  $A_{-i}^* = [-1, 1]$ .

[Part 4](#): By [Lemma A.8](#) there is no equilibrium in which both candidates win with positive probability.  $L$  cannot win with probability one because  $R$  can choose a platform that guarantees winning, as  $W_0(A_R^0, R) > W_0(A_L^0, L)$  by [part 2 of Proposition 2](#). So let  $A_R^\delta \in \mathcal{A}_R^{PO}$  denote  $R$ 's equilibrium platform that wins with probability one. It cannot be that  $W_0(A_R^\delta, R) > W_0(A_L^0, L)$ , because then  $\delta < 1$  (as by hypothesis  $W_0(A_R^1, R) < W_0(A_L^0, L)$ ) and  $R$  can do better by choosing a slightly higher  $\delta$  and still winning. It also cannot be that  $W_0(A_R^\delta, R) < W_0(A_L^0, L)$  because then  $L$  can do better by playing  $A_L^0$  and winning. Thus,  $L$  must be playing  $A_L^0$  and  $R$ 's platform must leave the voter indifferent. So  $R$ 's platform is  $[\alpha_R(-1), \bar{a}_R^*]$  where  $\bar{a}_R^*$  is the

solution to

$$\max_{\bar{a}_R \in [\bar{a}_0, \alpha_R(1)]} W_R([\alpha_R(-1), \bar{a}_R], R) \text{ s.t. } W_0([\alpha_R(-1), \bar{a}_R], R) = W_0(A_L^0, L).$$

Since the maximand is increasing in  $\bar{a}_R$  on the relevant domain, the uniqueness of  $\bar{a}_R^*$  follows if  $W_0([\alpha_R(-1), \bar{a}_R], R)$  is decreasing on  $\bar{a}_R \in [\bar{a}_0, \alpha_R(1)]$ . On this domain,

$$W_0([\alpha_R(-1), \bar{a}_R], R) = - \int_{-1}^{\bar{a}_R - b_R} b_R^2 f(\theta) d\theta - \int_{\bar{a}_R - b_R}^1 (\theta - \bar{a}_R)^2 f(\theta) d\theta,$$

and hence

$$\begin{aligned} \frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} &= 2 \int_{\bar{a}_R - b_R}^1 (\theta - \bar{a}_R) f(\theta) d\theta \\ &= 2(1 - F(\bar{a}_R - b_R)) \mathbb{E}[\theta - \bar{a}_R | \theta \geq \bar{a}_R - b_R]. \end{aligned} \quad (13)$$

Since  $\mathbb{E}[\theta - \bar{a}^0 | \theta \geq \bar{a}_0 - b_R] = 0$  by definition of  $\bar{a}_0$  (recall (4)), condition (3) implies that expression (13) is negative for all  $\bar{a}_R \in (\bar{a}_0, \alpha_R(1))$ .

**Part 5:** Since  $W_0(A_R^1, R) > W(A_L^0, L)$  by hypothesis, it is immediate that in any equilibrium  $R$  must win with probability one by choosing  $A_R^* = A_R^1$ .

## A.5. Details on Implications

As noted in the text, all the implications in [Section 4](#) are stated for cases where for each  $i \in \{L, R\}$ ,  $W^0(A_i^0, i) > W(A_{-i}^1, -i)$  and  $|b_i| > 0$ ; in other words, setting aside the cases that fall into parts 2 and 5 of [Proposition 3](#). With this in mind, we enumerate each implication below and explain how it derives from each part of [Proposition 3](#):

1. Both candidates propose the same platform in part 1 but not in parts 3 and 4.
2. Candidates are extreme in part 1. The case of part 3 follows from [Proposition 2](#) part 3. For part 4, it holds *a fortiori*.
3. Neither candidate is ambiguous in part 1. In part 3, the symmetry of  $f(\cdot)$  and that  $\underline{a}^0 = \bar{a}^0$  implies that they are equally ambiguous. So the only relevant case is part 4. Here, the result is immediate if  $-b_L > 1$  because then  $L$  is not ambiguous; the case of  $b_L < 1$  follows from [Lemma A.9](#) below.
4. The relevant case is part 3, and the result follows from [Proposition 2](#) part 3.

5. Divergence occurs in parts 3 and 4. In both cases, voter welfare is  $W_0(A_L^0, L)$ , which is higher than  $W_0(\{0\}, L)$ , the highest possible welfare under singleton platforms.
6. In parts 1, 3 and 4, candidates are less ambiguous than they would be without competition (in which case they would choose their full-discretion platform). However only in parts 1 and 3 does the winning candidate choose his voter-optimal platform.
7. With symmetric bias, this result follows from part 3 and Proposition 2 part 2. For the asymmetric case, part 4 and Proposition 2 part 2 imply that voter welfare increases when  $L$ 's bias decreases in magnitude and is constant in  $R$ 's bias, as voter welfare equals  $W_0(A_L^0, L)$  even though  $R$  wins with probability one.

It remains only to establish the following lemma.

**Lemma A.9.** *If  $0 < b_R < -b_L < 1$  and electoral competition is effective then  $R$  is more ambiguous than  $L$ .*

**Proof.** Fix any  $0 < b_R < -b_L < 1$ . We will show below that given any policy platform  $[\alpha_R(-1), \bar{a}_R]$  with  $\bar{a}_R \in [\bar{a}_0, \alpha_R(1)]$ , the variance of  $R$ 's policy is independent of  $b_R$  and increasing in  $\bar{a}_R$ . This fact implies the lemma when combined with the symmetric analogue for candidate  $L$  and that by part 4 of Proposition 3,  $\bar{a}_R^* > \bar{a}^0 > -\underline{a}_0 = -\underline{a}_L^*$ .

To prove the claim, it is useful to denote  $\hat{\theta}_R := \bar{a}_R - b_R$ . Since  $R$ 's policy in any state  $\theta$  is  $\min\{\theta + b_R, \hat{\theta}_R + b_R\}$ ,  $R$ 's expected policy is

$$\mathbb{E}[a_R(\theta, [\alpha_R(-1), \bar{a}_R])] = \int_{-1}^{\hat{\theta}_R} (\theta + b_R) f(\theta) d\theta + (1 - F(\hat{\theta}_R))(\hat{\theta}_R + b_R) = b_R + a_R^e(\hat{\theta}_R),$$

where  $a_R^e(\hat{\theta}_R) := \int_{-1}^{\hat{\theta}_R} \theta f(\theta) d\theta + (1 - F(\hat{\theta}_R))\hat{\theta}_R$ .

Hence, the variance of  $R$ 's policy, which we denote  $V_R(\hat{\theta}_R)$ , is

$$\begin{aligned} V_R(\hat{\theta}_R) &= \int_{-1}^1 (a_R(\theta, [\alpha_R(-1), \bar{a}_R]) - b_R - a_R^e(\hat{\theta}_R))^2 f(\theta) d\theta \\ &= \int_{-1}^{\hat{\theta}_R} (\theta - a_R^e(\hat{\theta}_R))^2 f(\theta) d\theta + (1 - F(\hat{\theta}_R))(\hat{\theta}_R - a_R^e(\hat{\theta}_R))^2. \end{aligned}$$

So  $V_R(\hat{\theta}_R)$  is independent of  $b_R$ . Differentiating,

$$V_R'(\hat{\theta}_R) = -2 \frac{\partial a_R^e(\hat{\theta}_R)}{\partial \hat{\theta}_R} \int_{-1}^{\hat{\theta}_R} (\theta - a_R^e(\hat{\theta}_R)) f(\theta) d\theta + 2 \left( 1 - \frac{\partial a_R^e(\hat{\theta}_R)}{\partial \hat{\theta}_R} \right) (1 - F(\hat{\theta}_R))(\hat{\theta}_R - a_R^e(\hat{\theta}_R)).$$

The RHS above is positive for all  $\hat{\theta}_R \in (-1, 1)$  because in this domain  $\frac{\partial a_R^e(\hat{\theta}_R)}{\partial \hat{\theta}_R} = 1 - F(\hat{\theta}_R) \in (0, 1)$ ,  $\hat{\theta}_R > a_R^e(\hat{\theta}_R)$ , and  $\int_{-1}^{\hat{\theta}_R} (\theta - a_R^e(\hat{\theta}_R)) f(\theta) d\theta = (1 - F(\hat{\theta}_R))(a_R^e(\hat{\theta}_R) - \hat{\theta}_R) < 0$ .  $\square$

## A.6. Proof of Proposition 4

Fix any  $A_i$  and  $A_j$ . Writing  $W_v(\cdot)$  for the welfare of voter  $v$ , we compute

$$W_v(A_i, i) - W_v(A_j, j) = \int_{-1}^1 [(a_j(\theta, A_j) - v - \theta)^2 - (a_i(\theta, A_i) - v - \theta)^2] f(\theta) d\theta.$$

Differentiating and simplifying yields

$$\frac{\partial [W_v(A_i, i) - W_v(A_j, j)]}{\partial v} = -2 \int_{-1}^1 (a_j(\theta, A_j) - a_i(\theta, A_i)) f(\theta) d\theta,$$

which is independent of  $v$ . Thus, if  $W_0(A_i, i) - W_0(A_j, j) > 0$ , then either  $W_v(A_i, i) - W_v(A_j, j) > 0$  for all  $v \leq 0$  or for all  $v \geq 0$ . Conversely, if  $W_0(A_i, i) - W_0(A_j, j) \leq 0$ , then either  $W_v(A_i, i) - W_v(A_j, j) \leq 0$  for all  $v \leq 0$  or for all  $v \geq 0$ .

## A.7. Proof of Proposition 5

**Part 1:** We claim that in any pure-strategy equilibrium with platforms  $A_L^*$  and  $A_R^*$ , it holds that  $A_i^* \in \mathcal{A}_i^{PO}$  for each  $i \in \{L, R\}$ , where  $\mathcal{A}_i^{PO}$  was defined in Proposition A.1. To see why the claim is true, observe that each candidate must win with positive probability (owing to the support of the valence shock; see fn. 21) and that  $W_i(A_i^*, i) > W_i(A_{-i}^*, -i)$  for each  $i \in \{L, R\}$ ; otherwise,  $i$  could deviate to proposing  $[\alpha_i(-1), \alpha_i(1)]$  and increase his expected utility. Consequently, the same logic as in Lemma A.3 applies.

So, given the hypothesis  $|b_i| \in (0, 1)$  for each  $i \in \{L, R\}$ , the platforms in any pure-strategy equilibrium satisfy  $A_L^* = [\underline{a}_L^*, \alpha_L(1)]$ , with  $\underline{a}_L^* \in [\alpha_L(-1), \underline{a}^0]$ , and  $A_R^* = [\alpha_R(-1), \bar{a}_R^*]$ , with  $\bar{a}_R^* \in [\bar{a}^0, \alpha_R(1)]$ . It remains to prove that  $\bar{a}_R^* \notin \{\bar{a}^0, \alpha_R(1)\}$ ; the argument for  $L$  is analogous.

Given any  $A_L$ ,  $R$ 's optimal choice of  $\bar{a}_R$  for a platform  $[\alpha_R(-1), \bar{a}_R]$  maximizes

$$\widehat{W}_R(\bar{a}_R, A_L) := G(\Delta_0(\bar{a}_R, A_L)) \Delta_R(\bar{a}_R, A_L) + W_R(A_L, L), \quad (14)$$

where for  $i \in \{L, R, 0\}$ ,  $\Delta_i(\bar{a}_R, A_L) := W_i([\alpha_R(-1), \bar{a}_R], R) - W_i(A_L, L)$ . Differentiating,

$$\frac{\partial \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R} = G(\Delta_0(\cdot)) \frac{\partial W_R(\cdot)}{\partial \bar{a}_R} + g(\Delta_0(\cdot)) \frac{\partial W_0(\cdot)}{\partial \bar{a}_R} \Delta_R(\cdot), \quad (15)$$

$$\begin{aligned} \frac{\partial^2 \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R^2} &= G(\Delta_0(\cdot)) \frac{\partial^2 W_R(\cdot)}{\partial \bar{a}_R^2} + g(\Delta_0(\cdot)) \frac{\partial \Delta_0(\cdot)}{\partial \bar{a}_R} \frac{\partial W_R(\cdot)}{\partial \bar{a}_R} \\ &\quad + \frac{\partial \{g(\Delta_0(\cdot)) \Delta_R(\cdot)\}}{\partial \bar{a}_R} \frac{\partial W_0(\cdot)}{\partial \bar{a}_R} + \frac{\partial^2 W_0(\cdot)}{\partial \bar{a}_R^2} g(\Delta_0(\cdot)) \Delta_R(\cdot). \end{aligned} \quad (16)$$

Note that on the relevant domain,  $\bar{a}_R \in [\bar{a}^0, \alpha_R]$ ,  $G(\cdot) > 0$  and  $g(\cdot) > 0$ . Using

$$W_R([\alpha_R(-1), \bar{a}_R], R) = - \int_{\bar{a}_R - b_R}^1 (\bar{a}_R - b_R - \theta)^2 f(\theta) d\theta$$

and recalling Equation 13, we compute

$$\frac{\partial W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} = -2 \int_{\bar{a}_R - b_R}^1 (\bar{a}_R - b_R - \theta) f(\theta) d\theta, \quad (17)$$

$$\frac{\partial^2 W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} = -2(1 - F(\bar{a}_R - b_R)), \quad (18)$$

$$\frac{\partial^2 W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} = 2b_R f(\bar{a}_R - b_R) - 2(1 - F(\bar{a}_R - b_R)). \quad (19)$$

Equation 13 and Equation 17 imply

$$\left. \frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \right|_{\bar{a}_R = \bar{a}^0} = 0 < \left. \frac{\partial W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \right|_{\bar{a}_R = \bar{a}^0},$$

so that the RHS of Equation 15 is positive at  $\bar{a}_R = \bar{a}^0$ , which implies  $\bar{a}_R^* > \bar{a}^0$ .

Equation 13, Equation 17, Equation 18, and Equation 19 imply

$$\begin{aligned} \left. \frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \right|_{\bar{a}_R = \alpha_R(1)} &= \left. \frac{\partial W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \right|_{\bar{a}_R = \alpha_R(1)} = 0, \\ \left. \frac{\partial^2 W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \right|_{\bar{a}_R = \alpha_R(1)} &= 0 < \left. \frac{\partial^2 W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \right|_{\bar{a}_R = \alpha_R(1)}. \end{aligned}$$

Hence, at  $\bar{a}_R = \alpha_R(1)$ , the RHS of Equation 15 is zero while the RHS of Equation 16 is positive (noting that  $\Delta_R(\alpha_R(1), A_L) > 0$ ). Thus,  $\widehat{W}_R(\cdot, A_L)$  is strictly convex in a neighborhood  $\alpha_R(-1)$  and attains a local minimum at  $\alpha_R(-1)$ , which implies  $\bar{a}_R^* < \alpha_R(1)$ .

**Part 2:** We prove the result for  $i = R$ ; it is symmetric for  $i = L$ . For any given  $\bar{a}_R \in (\bar{a}^0, \alpha_R(1))$  and any  $A_L$ , each term in (15) is independent of  $\phi$ , except  $\Delta_R([\alpha_R(-1), \bar{a}_R], R)$ , which tends to  $\infty$  as  $\phi \rightarrow \infty$ . Moreover,  $\frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} < 0$  for any  $\bar{a}_R \in (\bar{a}^0, \alpha_R(1))$ , as was established in the discussion around Equation 13. Thus, for any value of  $\bar{a}_R \in (\bar{a}^0, \alpha_R(1))$ , the

RHS of (15) becomes negative as  $\phi \rightarrow \infty$ , and we established above that  $\bar{a}_R = \alpha_R(1)$  is a local minimizer of  $\widehat{W}_R(\bar{a}_R, A_L)$ . The result follows.

## A.8. Proof of Proposition 6

Candidate  $R$  wins the election if and only if  $\eta > W_0(A_L^*, L) - W_0(A_R^*, R)$ . By part 1 of Proposition 2 and Lemma A.9,  $R$  is more ambiguous if elected under  $A_R^0$  than  $L$  is if elected under  $A_L^0$ . By part 2 of Proposition 2,  $W_0(A_L^0, L) < W_0(A_R^0, R)$ . Hence, by part 2 of Proposition 5, there exists a  $\bar{\phi}$  such that for all  $\phi > \bar{\phi}$ , (i)  $R$ 's platform is more ambiguous than  $L$ 's, and (ii)  $R$  is elected with probability  $G(W_0(A_R^*, R) - W_0(A_L^*, L)) > 1/2$ .

## B. Existence of Equilibrium with Valence Shocks

In this appendix, we consider the setting with valence shocks described in Subsection 5.3. We first provide sufficient conditions for the existence of a pure-strategy equilibrium.

**Proposition B.1.** *Assume  $|b_i| \in (0, 1)$  for each  $i \in \{L, R\}$  and let  $\eta$  be uniformly distributed on  $[-\lambda, \lambda]$ . For all  $\lambda$  large enough, a pure-strategy equilibrium exists.*

**Proof.** Given any  $A_L$ , the logic used in the proof of Proposition 5 (part 1) implies that any best response  $A_R$  is an interval  $A_R = [\alpha_R(-1), \bar{a}_R]$  for some  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$ . As the analog holds for candidate  $L$ , it suffices to show that there is a pure-strategy equilibrium when the candidates are restricted to use interval platforms with respective thresholds  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$  and  $\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]$ . Restricting to such platforms defines a game with non-empty, compact, convex one-dimensional strategy spaces, where candidate  $R$ 's payoff is given by  $\widehat{W}_R(\bar{a}_R, [\underline{a}_L, \alpha_L(1)])$  defined in Equation 14, and candidate  $L$ 's payoff is analogous. Since  $W_R(\bar{a}_R, [\underline{a}_L, \alpha_L(1)])$  is continuous in both  $\bar{a}_R$  and  $\underline{a}_L$ , it suffices to show that  $W_R$  is quasi-concave in  $\bar{a}_R$ , as a standard existence argument (based, for example, on Kakutani's fixed point theorem) then guarantees pure-strategy equilibria (using a symmetric argument for quasi-concavity of  $L$ 's payoff).

Accordingly, fix any  $A_L = [\alpha_L(-1), \underline{a}_L]$  with  $\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]$ ; we will establish that a large enough  $\lambda$  assures quasi-concavity of  $\widehat{W}_R(\bar{a}_R, A_L)$ .<sup>24</sup> The proof of Proposition 5 (Part 1) showed that  $\widehat{W}_R(\bar{a}_R, A_L)$  attains a local minimum at both  $\bar{a}_R = \bar{a}^0$  and  $\bar{a}_R = \alpha_R(1)$ , being strictly increasing at the former and strictly decreasing at the latter. Thus, if  $\frac{\partial^3 \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R^3} > 0$  for all  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$ , then  $\widehat{W}_R(\cdot, A_L)$  is strictly quasi-concave.

<sup>24</sup> Since the set of such  $A_L$  is compact, it follows that a large enough  $\lambda$  assures quasi-concavity of  $\widehat{W}_R(\bar{a}_R, A_L)$  in  $\bar{a}_R$  for all such  $A_L$ .

Plugging in the uniform distribution for  $\eta$  into Equation 16 yields:

$$\begin{aligned} \frac{\partial^2 \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R^2} &= \left[ \frac{1}{2} + \frac{W_0([\alpha_R(-1), \bar{a}_R], R) - W_0(A_L, L)}{2\lambda} \right] \frac{\partial^2 W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \\ &\quad + \frac{1}{\lambda} \frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \frac{\partial W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \\ &\quad + \frac{1}{2\lambda} \frac{\partial^2 W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \left[ W_R([\alpha_R(-1), \bar{a}_R], R) - W_R(A_L, L) \right]. \end{aligned} \quad (20)$$

Differentiating Equation 20 and plugging in derivatives of Equation 18 and Equation 19:

$$\begin{aligned} \frac{\partial^3 \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R^3} &= \left[ 1 + \frac{W_0([\alpha_R(-1), \bar{a}_R], R) - W_0(A_L, L)}{\lambda} \right] f(\bar{a}_R - b_R) \\ &\quad + \frac{3}{2\lambda} \frac{\partial^2 W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \frac{\partial W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \\ &\quad + \frac{3}{2\lambda} \frac{\partial W_0([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R} \frac{\partial^2 W_R([\alpha_R(-1), \bar{a}_R], R)}{\partial \bar{a}_R^2} \\ &\quad + \frac{1}{\lambda} [b_R f'(\bar{a}_R - b_R) + f(\bar{a}_R - b_R)] [W_R([\alpha_R(-1), \bar{a}_R], R) - W_R(A_L, L)]. \end{aligned}$$

As  $\lambda \rightarrow \infty$ , the RHS above converges to  $f(\bar{a}_R - b_R) > 0$ , where the inequality is because  $-1 \leq \bar{a}^0 - b_R \leq \bar{a}_R - b_R \leq \bar{\alpha}_R(1) - b_R = 1$  and  $f(\cdot) > 0$  on  $[-1, 1]$ . Consequently,  $\frac{\partial^3 \widehat{W}_R(\bar{a}_R, A_L)}{\partial \bar{a}_R^3} > 0$  for all  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$  once  $\lambda$  is large enough.  $\square$

We next show that once we allow for mixed strategies, an equilibrium always exists and that every equilibrium must involve candidates competing in interval platforms; moreover, as the concern for holding office gets arbitrarily large, candidates' platforms converge to the voter-optimal platform.

**Proposition B.2.** *Suppose  $|b_i| \in (0, 1)$  for each  $i \in \{L, R\}$ . Then a (possibly-mixed) equilibrium exists. Furthermore:*

1. *In every equilibrium, with probability one, candidate L's platform is in*

$$\mathcal{A}_L^{PO} = \{[\underline{a}_L, \alpha_L(1)] : \underline{a}_L \in [\alpha_L(-1), \underline{a}^0]\},$$

*and candidate R's platform is in*

$$\mathcal{A}_R^{PO} = \{[\alpha_R(-1), \bar{a}_R] : \bar{a}_R \in [\bar{a}^0, \alpha_R(1)]\}.$$



2. For any  $\varepsilon > 0$ , there exists a  $\bar{\phi}$  such that, for all  $\phi > \bar{\phi}$ , in every equilibrium, with probability one, candidate  $L$ 's platform is in  $\{[\underline{a}_L, \alpha_L(1)] : \underline{a}_L \in [\underline{a}^0 - \varepsilon, \underline{a}^0]\}$ , and candidate  $R$ 's platform is in  $\{[\alpha_R(-1), \bar{a}_R] : \bar{a}_R \in [\bar{a}^0, \bar{a}^0 + \varepsilon]\}$ .

**Proof.** The argument for part 1 is the same as in the [proof of Proposition 5](#) (part 1). Hence we can restrict attention to each candidate  $i$  choosing among platforms  $\mathcal{A}_i^{PO}$ , or equivalently choosing among the corresponding thresholds,  $\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]$  and  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$  respectively. As these strategy spaces are non-empty, compact, one-dimensional sets and payoffs are continuous (as discussed in the [proof of Proposition B.1](#)), existence of equilibrium follows from [Glicksberg \(1952\)](#).

We prove part 2 of the proposition for  $i = R$ ; the argument is symmetric for  $i = L$ . By part 1, we can restrict attention to platforms of the form  $A_L = [\underline{a}_L, \alpha_L(1)]$  and  $A_R = [\alpha_R(-1), \bar{a}_R]$  with  $\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]$  and  $\bar{a}_R \in [\bar{a}^0, \alpha_R(1)]$ . Recall that by definition

$$\Delta_i(\bar{a}_R, A_L) = W_i([\alpha_R(-1), \bar{a}_R], R) - W_i(A_L, L).$$

For any  $\bar{a}_R \in (\bar{a}^0, \alpha_R(1)]$  the following statements hold for all  $\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]$ :

1.  $W_0([\alpha_R(-1), \bar{a}_R], R) < W_0([\alpha_R(-1), \bar{a}^0], R)$ , and so  
 $G(\Delta_0(\bar{a}_R, [\underline{a}_L, \alpha_L(1)])) < G(\Delta_0(\bar{a}^0, [\underline{a}_L, \alpha_L(1)]));$
2.  $\Delta_R(\bar{a}_R, [\underline{a}_L, \alpha_L(1)]) - \Delta_R(\bar{a}^0, [\underline{a}_L, \alpha_L(1)])$  is bounded;
3.  $\Delta_R(\bar{a}^0, [\underline{a}_L, \alpha_L(1)]) \geq \phi$ .

As  $[\alpha_L(-1), \underline{a}^0]$  is compact and  $\widehat{W}_R(\bar{a}_R, \underline{a}_L)$  defined in [Equation 14](#) is continuous, it follows that for any  $\bar{a}_R$ ,

$$\lim_{\phi \rightarrow \infty} \min_{\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]} \widehat{W}_R(\bar{a}^0, \underline{a}_L) - \widehat{W}_R(\bar{a}_R, \underline{a}_L) = \infty.$$

Therefore, for any  $\varepsilon > 0$ , there exists a  $\bar{\phi}$  such that, for all  $\phi > \bar{\phi}$  and  $\bar{a}_R \in [\bar{a}^0 + \varepsilon, \alpha_R(1)]$ ,

$$\min_{\underline{a}_L \in [\alpha_L(-1), \underline{a}^0]} \widehat{W}_R(\bar{a}^0, \underline{a}_L) - \widehat{W}_R(\bar{a}_R, \underline{a}_L) > 0.$$

It follows that when  $\phi > \bar{\phi}$ , candidate  $R$  cannot be playing a best response unless he chooses some  $\bar{a}_R \in [\bar{a}^0, \bar{a}^0 + \varepsilon]$  with probability one.  $\square$

Finally, notice that part 2 of [Proposition B.2](#) implies that, with high probability, the more moderate candidate proposes the more ambiguous platform and wins with probability greater than 1/2. Consequently, one can derive a mixed-strategy analogue of [Proposition 6](#) as well.

## References

- ALESINA, A. AND A. CUKIERMAN (1990): "The Politics of Ambiguity," *Quarterly Journal of Economics*, 105, 829–850.
- ALONSO, R. AND N. MATOUSCHEK (2008): "Optimal Delegation," *Review of Economic Studies*, 75, 259–293.
- AMADOR, M. AND K. BAGWELL (2013): "The Theory of Optimal Delegation with an Application to Tariff Caps," *Econometrica*, 81, 1541–1599.
- AMADOR, M., I. WERNING, AND G. M. ANGELETOS (2006): "Commitment vs. Flexibility," *Econometrica*, 74, 365–396.
- AMBRUS, A., V. BARANOVSKYI, AND A. KOLB (2015): "A Delegation-Based Theory of Expertise," Duke University, mimeo.
- ARAGONES, E. AND Z. NEEMAN (2000): "Strategic Ambiguity in Electoral Competition," *Journal of Theoretical Politics*, 12, 183–204.
- ARAGONES, E. AND A. POSTLEWAITE (2002): "Ambiguity in Election Games," *Review of Economic Design*, 7, 233–255.
- AZZIMONTI, M. (2011): "Barriers to Investment in Polarized Societies," *American Economic Review*, 101, 2182–2204.
- BAGNOLI, M. AND T. BERGSTROM (2005): "Log-concave Probability and its Applications," *Economic Theory*, 26, 445–469.
- BENDOR, J. AND A. MEIROWITZ (2004): "Spatial Models of Delegation," *American Political Science Review*, 98, 293–310.
- BERENSKY, A. AND J. B. LEWIS (2007): "An Estimate of Risk Aversion in the U.S. Electorate," *Quarterly Journal of Political Science*, 2, 139–154.
- BERLIANT, M. AND H. KONISHI (2005): "Salience: Agenda Choices by Competing Candidates," *Public Choice*, 125, 129–149.
- BERNHARDT, D. M., J. DUGGAN, AND F. SQUINTANI (2009): "The Case for Responsible Parties," *American Political Science Review*, 103, 570–587.
- CALLANDER, S. AND C. WILSON (2008): "Context-dependent Voting and Political Ambiguity," *Journal of Public Economics*, 92, 565–581.
- CALVERT, R. L. (1985): "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, 29, 69–95.
- CONDON, S. (2012): "Conservatives Worry Romney's Agenda is too Vague," *CBS News*, September 25.

- DOWNES, A. (1957): *An Economic Theory of Democracy*, New York: Harper and Row.
- EPSTEIN, D. AND S. O'HALLORAN (1994): "Administrative Procedures, Information, and Agency Discretion," *American Journal of Political Science*, 38, 697–722.
- FIORINA, M. P., S. J. ABRAMS, AND J. C. POPE (2010): *Culture War? The Myth of a Polarized America*, Pearson Longman New York.
- FOX, J. AND K. W. SHOTTS (2009): "Delegates or Trustees? A Theory of Political Accountability," *Journal of Politics*, 72, 1225–1237.
- FRENKEL, S. (2014): "Competence and Ambiguity in Electoral Competition," *Public Choice*, 159, 219–234.
- GAILMARD, S. (2009): "Discretion Rather than Rules: Choice of Instruments to Control Bureaucratic Policymaking," *Political Analysis*, 17, 25–44.
- GAILMARD, S. AND J. W. PATTY (2012): "Formal Models of Bureaucracy," *Annual Review of Political Science*, 15, 353–377.
- GILLIGAN, T. W. AND K. KREHBIEL (1987): "Collective Decisionmaking and Standing Committees: An Informational Rationale for Restrictive Amendment Procedures," *Journal of Law, Economics, & Organization*, 3, 287–335.
- GLAZER, A. (1990): "The Strategy of Candidate Ambiguity," *American Political Science Review*, 84, 237–241.
- GLICKSBERG, I. L. (1952): "A Further Generalization of the Kakutani Fixed Point Theorem, with Application to Nash Equilibrium Points," *Proceedings of the American Mathematical Society*, 3, 170–174.
- HEIDHUES, P. AND J. LAGERLOF (2003): "Hiding Information in Electoral Competition," *Games and Economic Behavior*, 42, 48–74.
- HOLMSTRÖM, B. (1977): "On Incentives and Control in Organizations," Ph.D. Thesis, Stanford University.
- (1984): "On the Theory of Delegation," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. Kihlstrom, North Holland, 115–141.
- JENSEN, T. (2009): "Projection Effects and Strategic Ambiguity in Electoral Competition," *Public Choice*, 141, 213–232.
- KAMADA, Y. AND T. SUGAYA (2014): "Valence Candidates and Ambiguous Platforms in Policy Announcement Games," University of California Berkeley, mimeo.
- KARTIK, N., F. SQUINTANI, AND K. TINN (2014): "Information Revelation and Pandering in Elections," Columbia University, mimeo.

- KNIGHT, F. H. (1921): *Risk, Uncertainty and Profit*, Boston: Houghton Mifflin.
- LEE, D. S., E. MORETTI, AND M. J. BUTLER (2004): "Do Voters Affect or Elect Policies? Evidence from the US House," *Quarterly Journal of Economics*, 807–859.
- LIZZA, R. (2012): "Obama Goes Vague," *The New Yorker*, September 7.
- MARTINELLI, C. AND A. MATSUI (2002): "Policy Reversals and Electoral Competition with Privately Informed Parties," *Journal of Public Economic Theory*, 4, 39–61.
- MEIROWITZ, A. (2005): "Informational Party Primaries and Strategic Ambiguity," *Journal of Theoretical Politics*, 17, 107–136.
- MELUMAD, N. AND T. SHIBANO (1991): "Communication in Settings with No Transfers," *RAND Journal of Economics*, 22, 173–198.
- OBAMA, B. (2008): "Remarks in Dover, New Hampshire," Available at The American Presidency Project.
- PAGE, B. (1976): "The Theory of Political Ambiguity," *American Political Science Review*, 70, 742–752.
- SAHADI, J. (2012): "Social Security: Where Obama and Romney Stand," *CNN Money*, October 10.
- SHEPSLE, K. A. (1972): "The Strategy of Ambiguity: Uncertainty and Electoral Competition," *American Political Science Review*, 66, 555–568.
- TANNER, N. (2013): "Optimal Delegation Under Uncertain Bias," Ph.D. Thesis, Yale University.
- TOMZ, M. AND R. P. VAN HOUWELING (2009): "The Electoral Implications of Candidate Ambiguity," *American Political Science Review*, 103, 83–98.
- URBINATI, N. AND M. E. WARREN (2008): "The Concept of Representation in Contemporary Democratic Theory," *Annual Review of Political Science*, 11, 387–412.
- VAN WEELDEN, R. (2013): "Candidates, Credibility, and Re-election Incentives," *Review of Economic Studies*, 80, 1622–1651.
- WESTERMARK, A. (2004): "Extremism, Campaigning and Ambiguity," *Games and Economic Behavior*, 47, 421–452.
- WISEMAN, A. E. (2009): "Delegation and Positive-Sum Bureaucracies," *Journal of Politics*, 71, 998–1014.
- WITTMAN, D. (1977): "Candidates with Policy Preferences," *Journal of Economic Theory*, 14, 180–189.

——— (1983): "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 77, 142–157.

ZECKHAUSER, R. (1969): "Majority Rule with Lotteries on Alternatives," *The Quarterly Journal of Economics*, 83, 696–703.